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Improving the performances of Earth Air Heat Exchangers through Constructal design

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Abstract

Earth to Air Heat Exchanger (EAHE) is a well-known technique used to preheat or precool outdoor air before blowing it into a building. However, its geometry is often very simple as it consists in one or multiple straight pipes, while more complex arrangement can be found for heat exchangers. In this paper, we explore the advantage of designing an EAHE as a network by using the Constructal Law point of view. A methodology is first proposed to design a single pipe EAHE when the need is defined in terms of cooling power, overall efficiency and enthalpy difference between the inlet air and the ground. Next, the single pipe EAHE is used as a reference for designing a tree-shaped network under the constraint of identical fluid volume and cooling power. The geometrical features are allowed to change for the different branches of the network. The network coefficient of performance is found to increase significantly with the bifurcation level, illustrating the superior performances of the network. This approach was found to be robust as the improvements were not depending on the cooling demand or the environmental conditions. However, further work is needed to move from this analytical result to practical considerations.

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Keywords

Constructal Law; Dendritic Networks; Earth Air Heat Exchanger; Thermal efficiency;
Coefficient of Performance

Nomenclature

| Latin Symbols | Description | Unit |
|---------------|----------------------------------------|-------------------------------------------------|
| C_p | Specific heat | $\text{J}\cdot\text{kg}^{-1}\cdot\text{K}^{-1}$ |
| d | Inner diameter | m |
| f_D | Darcy Weisbach friction factor | - |
| h | Enthalpy | $\text{J}\cdot\text{kg}^{-1}$ |
| h_c | Convective heat transfer coefficient | $\text{W}\cdot\text{K}^{-1}\cdot\text{m}^{-2}$ |
| k | Thermal conductivity | $\text{W}\cdot\text{m}^{-1}\cdot\text{K}^{-1}$ |
| K | Constant | - |
| L | Length | m |
| \dot{m} | Mass flow rate | $\text{kg}\cdot\text{s}^{-1}$ |
| N | Number of bifurcations | |
| Nu | Nusselt number | - |
| p | Perimeter | m |
| P | Pressure | Pa |
| Pr | Prandtl number | - |
| \dot{Q} | Heat flux | W |
| Re_d | Reynolds number based on duct diameter | - |

| | | |
|-----------|-----------------------|--------------------|
| T | Temperature | $^{\circ}\text{C}$ |
| V | Volume | m^3 |
| x | Distance to the inlet | m |
| \dot{W} | Power of the fan | |

Greek Symbols

| | | |
|---------------|--------------------------------------------------|-------------------------------|
| ε | Coefficient of performance | - |
| ρ | Density | $\text{kg}\cdot\text{m}^{-3}$ |
| η | Dimensionless enthalpy at the outlet of the pipe | - |

Subscripts

| | |
|----|------------------------------|
| a | Air |
| ha | Humid air |
| in | Inlet |
| N | Number of bifurcation |
| w | Wall |
| wb | Wet bulb |
| 0 | Single pipe - no bifurcation |

Upperscripts

| | |
|--------|---------------|
| \sim | Dimensionless |
|--------|---------------|

1 INTRODUCTION

Earth Air Heat Exchangers (EAHE) can meet the challenge of replacing classical air conditioning systems by a more environmentally friendly solution, with the objective of decreasing energy consumption of buildings. An EAHE is composed of fans and buried

pipelines, so that the soil preheats or precools the air blown through the buried ducts, taking advantage of the thermal inertia of the earth[1, 2]. This is a well-known system that has already attracted a lot of interest in research. The interested reader is referred to [3] for a broad view on this topic, among others. While many research topics were identified for this technology, it was observed in [4] that the geometry of the EAHE was poorly studied and remained the same most of the time, that is, one or multiple parallel straight pipes buried at a constant depth. However, much complex arrangement can be found for similar systems. For example, complex 3D arrangements are often considered for ground source heat pumps [5], which uses a liquid as working fluid instead of air. This statement is the start-point of the present paper: we intend to examine if the geometry of a classical EAHE could be improved to enhance its performances. As this is a rather broad topic, we propose to study this problem through the global vision of Constructal design [6]. The Constructal law governs the occurrence and evolution of any type of flux architectures in nature and engineering towards configurations that facilitate flow access. In this work the underground network constituting the EAHE is considered as an evolving flow system. Therefore, the Constructal Law fits perfectly with the objective of the present work.

One of the first studies on the Constructal design of ground coupled heat pumps was published in 2012 [7]. The objective of the work was to determine the heat flow structure giving the maximum heat transfer between pipes and ground. In the same year, [8] studied the effect of the flow configuration on the thermal performance of a serpentine duct buried in the soil, while [9] covered the issue of underground heat exchangers for cooling at urban scale. Errera [10, 11] analyzed the coupling of an underground fluid loop to multiple heat pumps. Subsequently, studies presenting five different geometries used Constructal design to maximize the system thermal potential, i.e. the average of the differences between the air temperatures at the entrance

and the exit of the ducts [12]. More recently, the performance of different geometric constructs for EAHEs composed of four ducts has been evaluated [13] and a numerical analysis of the impact of the geometry, involving configurations from one to five ducts, has been developed in [14] with the objective of increasing the thermal potential of these devices.

The research presented in [15] focuses on the sensible and latent heat exchanges occurring when air is blown through an open underground system made of a single straight buried pipe. Accounting for the total enthalpy change rather than only the sensible heat exchanges highlights the impact of the environmental conditions (tropical vs. continental climates) on EAHE performance. The approach developed in this latest paper is extended in the present paper: here we intend to examine the benefit of using an underground network instead of a single buried pipe. The paper is made of two main parts: In the next section, an EAHE based on a single duct is considered. An analytical methodology is proposed to size the EAHE in summer conditions according to 4 constraints, chosen to make the EAHE competitive in terms of energy consumption when compared to a classical refrigeration system. The influence of the environmental conditions, namely the enthalpy of the outdoor air and of the soil, is briefly debated through a numerical application for typical French and Brazilian conditions. Next, in accordance with the Constructal law, the EAHE is designed as a network so that the air is blown through multiple outlets, while the cooling demand remains the same. Finally, the performance of the network is compared with that of a single pipe, and the influence of the environmental conditions and the cooling demand are highlighted.

2 EAHE MADE OF A SINGLE DUCT

The EAHE studied consists of a fan blowing air into a straight cylindrical duct. The exact position of the fan does not matter, as long as the electrical motor is located outside the duct. For

the sake of simplicity, the geometry of the system at the inlet and at the outlet is not considered in this model; the region of interest is the horizontal part of the EAHE: a cylinder of length L_0 and diameter d_0 .

2.1 Design of the EAHE

The equations of conservation of mass and energy were presented in [15] under the assumption of constant pipe wall temperature. As a result, Eq. (1) gives the evolution of the humid air enthalpy h within the pipe as a function of x , the distance to the inlet, assuming wet walls. This condition was chosen as it is often encountered in practice due to groundwater infiltrations. Note that it corresponds to a situation that is more challenging than dry walls in terms of thermal efficiency.

$$h(x) = (h_{in} - h_w) e^{-\frac{h_c p}{C_{p,ha} \dot{m}_a} x} + h_w \quad (1)$$

where h_{in} and h_w are the enthalpy of the humid air at the inlet and of the wall respectively, h_c is the convective heat transfer coefficient, $C_{p,ha}$ is the heat capacity of humid air, \dot{m}_a is the air flow rate and p is the perimeter of the pipe.

Next, we consider the dimensionless enthalpy \tilde{h} , given by:

$$\tilde{h}(x) = \frac{h(x) - h_w}{h_{in} - h_w} \quad (2)$$

The objective is to determine the diameter and length of the pipe consistent with a given cooling demand, while designing an energy efficient system. Related to these aspects, we define:

1. The cooling power \dot{Q} , i.e. the heat transferred from the air to the ground before the air enters the building. Note that the approach presented in this paper would be exactly the same if a heating power was required, except that the air would be heated by the ground;

2. The dimensionless enthalpy at the outlet of the pipe, $\tilde{h}(x = L_0)$, denoted η . The term $\tilde{h}(x)$ depends on the length of the pipe according to Eq. (1) and remains strictly positive, as illustrated in FIGURE 1. In this figure, η is identified for a length L_0 . Note that the definition of η is similar to the efficiency of heat exchangers, except that the ideal efficiency for heat exchangers is 1, while here the best performances would be obtained when η equals 0;
3. Consequently, the cooling power can be written :

$$\dot{Q} = (1 - \eta) \Delta h \dot{m}_a \quad (3)$$

where Δh is the enthalpy difference between the air at the inlet and the pipe, which is known for a given climate and period of the year. This term is positive for cooling purposes, and would be negative, in the case of heating.

$$\Delta h = h_{in} - h_w \quad (4)$$

The methodology proposed here considers a steady state. Therefore Δh must be considered as an average value over a given season (summer in this paper);

4. The coefficient of performance ε , which compares the cooling power to the power of the fan \dot{W} .

$$\varepsilon = \frac{\dot{Q}}{\dot{W}} \quad (5)$$

This definition is similar to the one used for classical refrigerating units, where the denominator represents the electric power of a compressor. Therefore, the value of ε for the EAHE should have the same order of magnitude as for a classical refrigerating unit (e.g. $3 < \varepsilon < 4$) in order to constitute a cost-effective alternative.

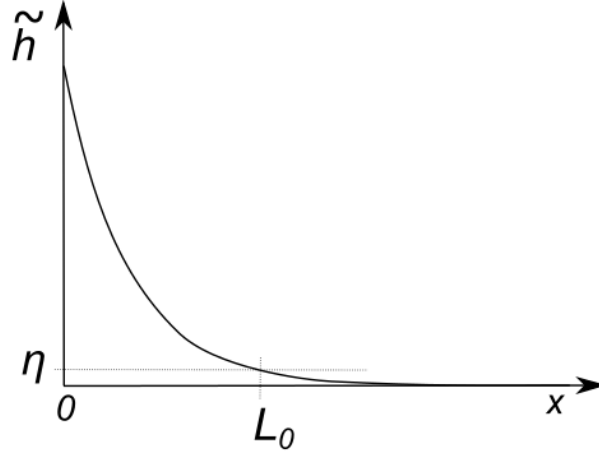


FIGURE 1. Dimensionless enthalpy at the outlet of a pipe.

Designing the EAHE involves discovering 3 parameters (L_0 , d_0 and \dot{m}_a) under the four constraints \dot{Q} , η , Δh and ε . From the definition of η and Eq. (1), we obtain:

$$\eta = \exp\left(-\frac{\pi h_c L_0}{C_{P,ha} \dot{m}_a} d_0\right) \quad (6)$$

$$L_0 = -\frac{C_{P,ha} \dot{m}_a \ln(\eta)}{\pi h_c d_0} \quad (7)$$

The power of the fan is related to the pressure drop ΔP , as shown in Eq. (8).

$$\dot{W} = \frac{\Delta P}{\rho_{ah}} \dot{m}_a \quad (8)$$

Neglecting the local pressure losses, ΔP is given by Eq. (9) in turbulent regime [16], where f_D is the Darcy-Weisbach friction factor.

$$\Delta P = f_D \frac{8 \dot{m}_a^2 L_0}{\rho_{ha} \pi^2 d_0^5} \quad (9)$$

Combining Eqs. (3), (5) and (8), we obtain another expression for the pressure drop,

$$\Delta P = \frac{(1 - \eta) \Delta h \rho_{ha}}{\varepsilon} \quad (10)$$

Combining Eqs. (9) and (10) leads to:

$$L_0 = \frac{(1 - \eta) \Delta h \pi^2 \rho_{ha}^2 d_0^5}{8 \varepsilon f_D \dot{m}_a^2} \quad (11)$$

The diameter is obtained by combining Eqs. (7) and (11):

$$d_0 = \left(-\frac{8 \dot{m}_a^3 f_D \varepsilon C_{p,ha} \ln(\eta)}{\rho_{ha}^2 \pi^3 (1 - \eta) \Delta h h_c} \right)^{\frac{1}{6}} \quad (12)$$

Finally, the mass flow rate is replaced by the means of (3), leading to the final expression for the pipe diameter.

$$d_0 = \left(-\frac{8 \dot{Q}^3 f_D \varepsilon C_{p,ha} \ln(\eta)}{\rho_{ha}^2 \pi^3 (1 - \eta)^4 \Delta h^4 h_c} \right)^{\frac{1}{6}} \quad (13)$$

The length of the pipe L_0 is finally given by Eq. (7), while the mass flow rate comes from Eq. (3). The originality of this approach is that the mass flow rate results from performance objectives instead of being imposed, as it is most commonly the case for the design of EAHEs.

2.2 Numerical Application

As mentioned earlier, the design of the EAHE depends on four parameters (\dot{Q} , η , Δh and ε). We propose to fix two of them for clarity, while investigating the others two.

- Considering $\eta = 0.05$ provides a good compromise between enough heat being exchanged between the air in the pipe and the soil and the total length of the EAHE remaining within reasonable limits, according to FIGURE 1. The same value of η was used in [15]. Note that the value of η can be set regardless of the other 3 parameters;

- For the EAHE to be cost-effective compared to a refrigerating unit, the coefficient of performance ε was set to 3.

In consequence, the numerical application focuses on the influence of the cooling demand, \dot{Q} , and on the total (sensible and latent) enthalpy difference Δh between the inlet and the pipe wall. Δh is somehow representative of the environmental conditions, although this study considers the steady state only.

The values for the convective heat transfer coefficient h_c and the friction factor f_D have not been addressed yet. Both depend on the pipe geometry, as shown in Eqs. (14) [16, 17], Eqs. (15) [17, 18] and Eqs. (16) [19]:

$$h_c = \frac{k Nu_d}{d} \quad (14)$$

$$Nu_d = \frac{\left(\frac{f_D}{8}\right)(Re_d - 10^3)Pr}{1 + 12.7\left(\frac{f_D}{8}\right)^{\frac{1}{2}}\left(Pr^{\frac{2}{3}} - 1\right)} \quad (15)$$

$$f_D = (0.79 \ln(Re_d) - 1.64)^{-2} \quad (16)$$

where the subscript $_d$ stands for the pipe diameter, Nu is the Nusselt number, Re is the Reynolds number, and Pr is the Prandtl number. Equations (12-14) show that d_0 , f_D and h_c depend on each other. Therefore an iterative procedure was used in the model that we developed, and convergence was obtained when the relative variation of the three values between two consecutive runs was lower than 10^{-4} .

Next, we propose to compute d_0 and L_0 for Δh ranging from 1 to 20 kJ.kg_a^{-1} , which is representative of realistic environmental conditions. The range of the cooling demand was

arbitrarily defined from 1 to 7 kW. A higher cooling demand would require the EAHE to be combined with another cooling system. The results are presented in FIGURE 2 and 3.

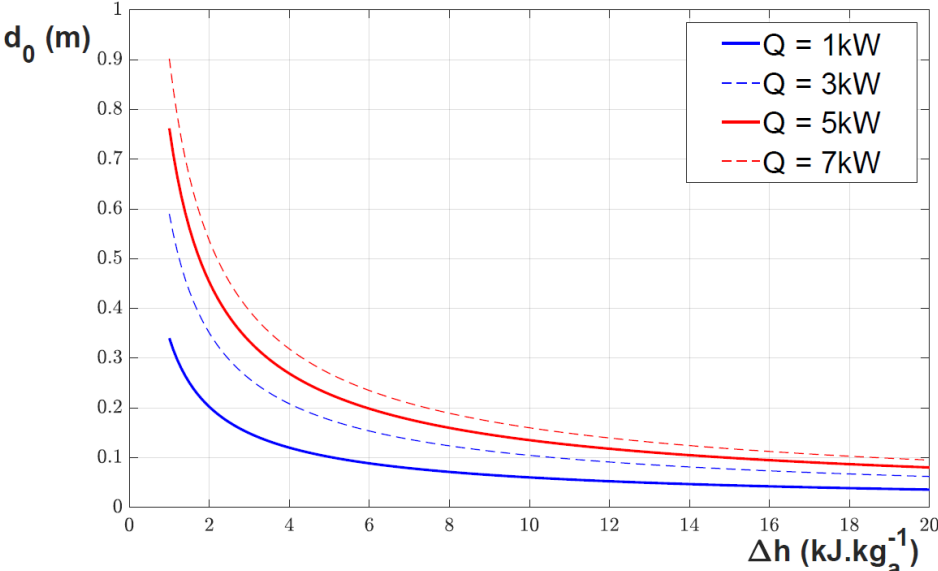


FIGURE 2. Pipe diameter as a function of the enthalpy difference ($\Delta h = h_{in} - h_w$) for different values of the targeted cooling power

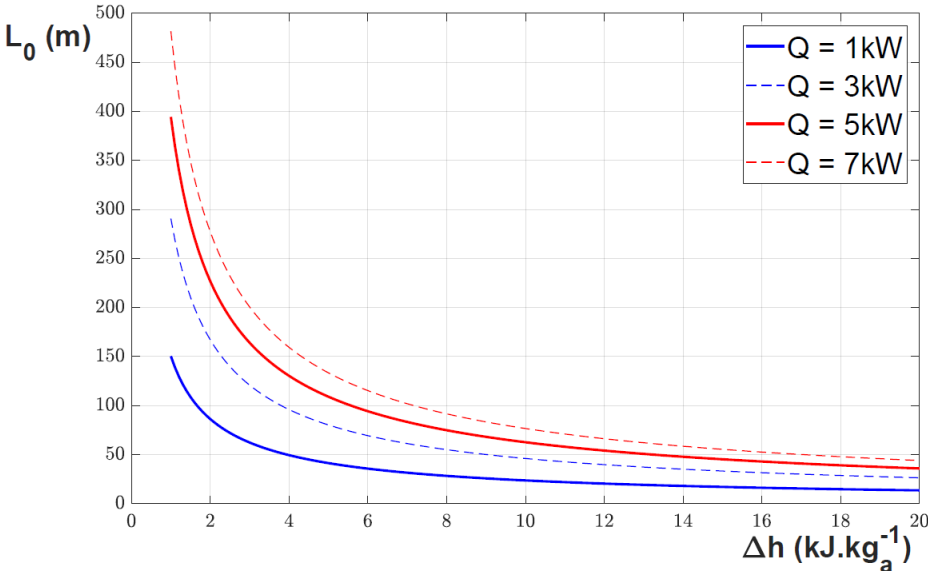


FIGURE 3. Pipe length as a function of the enthalpy difference ($\Delta\mathbf{h} = \mathbf{h}_{\text{in}} - \mathbf{h}_{\text{w}}$) for different values of the targeted cooling power

To analyze the shape of these curves, Eqs. (7), (9) and (13) were rearranged to highlight the influence of \dot{Q} and Δh .

$$L_0 = K_{L_0} \left(\frac{\dot{Q}^3}{\Delta h^2} \right)^{\frac{1}{6}} \quad (17)$$

$$\Delta P_0 = K_{P_0} \Delta h \quad (18)$$

$$d_0 = K_{d_0} \left(\frac{\dot{Q}^3}{\Delta h^4} \right)^{\frac{1}{6}} \quad (19)$$

where K_{L_0} , K_{P_0} and K_{d_0} depend on the other constraints and parameters (see Annex).

The above equations show that, unlike the pressure drop (see Eq. (19)), both the diameter and the length of the EAHE are functions of the square root of the cooling power, \dot{Q} . The impact of the enthalpy difference is different for d_0 , L_0 and ΔP_0 . Overall, a smaller enthalpy difference leads to a larger EAHE to obtain the same cooling power and efficiency. Note that the pressure drop varies linearly with Δh .

However, the 3 coefficients K_{d_0} , K_{L_0} and K_{P_0} depend on the friction factor f_D and on the convective heat transfer coefficient h_c . As mentioned earlier, these two coefficients are related to the diameter, the latter being a function of \dot{Q} and Δh . Thus, K_{d_0} , K_{L_0} and K_{P_0} depend on \dot{Q} and Δh . To go into greater details, the values of f_D and h_c were analyzed within the range of the numerical

application presented here (i.e. $1 \text{ kW} < \dot{Q} < 7 \text{ kW}$, and $1 \text{ kJ.kg}_a^{-1} < \Delta h < 20 \text{ kJ.kg}_a^{-1}$). The results are presented in Table 1.

Table 1 : Mean value and standard deviation for f_D , h_c and Re_d obtained when the cooling demand and the enthalpy difference vary over the range defined for the numerical application.

| Parameter | f_D | h_c W.m ⁻² .K ⁻¹ | Re_d |
|--------------------|--------|---------------------------------------------|------------------|
| Mean value | 0.0153 | 62 | $4.7 \cdot 10^6$ |
| Standard deviation | 0.013 | 21 | $6.0 \cdot 10^6$ |

Note that significant discrepancies were found for extreme configurations when the mean value was used instead of a precise one. For example, the diameter could be underestimated by 15% and the length by as much as 60%. We conclude that the exact values of f_D and h_c are necessary to design the EAHE precisely.

To give a more operational illustration of the design of the EAHE, we now propose to present the results for two different locations: Porto Alegre (Brazil) and Montpellier (France). The environmental conditions for summer months were extracted from [15] and are summarized in Table 2

Table 2 : Example of sizing for environmental conditions representative of a Brazilian and a French city.

| Location | T_{wb} °C | T_w °C | Δh kJ.kg _A ⁻¹ | \dot{Q} kW | d_0 cm | L_0 m |
|--------------|----------------|-------------|------------------------------------------------|-----------------|-------------|------------|
| Porto Alegre | 22 | 20 | 3 | 1 | 15 | 62.4 |
| | | | | 7 | 39.5 | 200.6 |
| Montpellier | 20 | 16 | 6 | 1 | 8.9 | 35.7 |
| | | | | 7 | 23.5 | 115.3 |

T_{wb} represents the wet bulb temperature, averaged over the summer period, blown at the pipe entrance, while T_w , the pipe wall temperature, is identical to the soil temperature at the depth where the pipe is implemented (3 m, in accordance with [15]). Montpellier benefits from a much larger temperature amplitude at the yearly scale, which results in a cooler soil in summer and favors the use of EAHE for cooling purposes. The average difference in summer Δh ($= h_{in} - h_w$) between the two places is mainly due to the latent contribution of the inlet enthalpy: the climate is hot and humid in summer in Porto Alegre, while it is hot and much drier in Montpellier.

Under the climate of Montpellier, a cooling power of 1 kW with a coefficient of performance $\varepsilon = 3$ can be reached with a pipe diameter of 8.9 cm and a length of 35.7 m. The same performance could be obtained in Porto Alegre at the cost of a larger pipe (diameter 15 cm) and a longer EAHE (62.4 m). Recalling that the value of Δh is 50% lower for Porto Alegre, obtaining the same performance requires the use of a pipe with a volume that is approximately 5 times bigger. This illustrates the significant influence of the outdoor conditions on the sizing of the EAHE. To boost the cooling power to 7 kW, the volume of the EAHE has to be multiplied by roughly 23 in order to obtain the same coefficient of performance, regardless of the value of Δh .

To give better insight into the influence of the EAHE dimensions (d_0 or L_0), we propose to determine the effect of a variation of 10% of these two parameters. Assuming that Δh and the mass flow rate remain the same, the variation of the cooling power and pressure drop can be obtained from Eqs. (6) and (9). The results are presented in Table 3.

Table 3 : Relative influence of a variation of the duct diameter and length on the cooling power, pipe volume and pressure drop.

| | $\frac{\dot{Q} _{d^*,L^*}}{\dot{Q} _{d_0,L_0}}$ | $\frac{\Delta P _{d^*,L^*}}{\Delta P _{d_0,L_0}}$ | $\frac{V _{d^*,L^*}}{V _{d_0,L_0}}$ |
|----------------------------|-------------------------------------------------|---------------------------------------------------|-------------------------------------|
| $d^* = 1.1 d_0, L^* = L_0$ | + 1.3% | - 38% | + 21% |
| $d^* = 0.9 d_0, L^* = L_0$ | - 1.8% | + 69% | - 19% |
| $d^* = d_0, L^* = 1.1 L_0$ | + 1.3% | + 10% | + 10% |
| $d^* = d_0, L^* = 0.9 L_0$ | - 1.8% | - 10% | - 10% |

These results show that marginal improvement on the cooling power could be obtained compared to the reference case, which is also a consequence of the very good value used for η (see FIGURE 1). However, the influence on the volume and on the pressure drop would be more significant. As the pipe diameter d_0 is to the power of five in Eq. (9), the pressure drop can be reduced by almost 40% by increasing d_0 by 10%. On the other hand, the effect of reducing L_0 is less pronounced. This leads us to the conclusion that a more efficient EAHE design could be obtained by considering a network instead of a single pipe.

3 DESIGN OF AN EAHE NETWORK WITH MULTIPLE OUTLETS

The Constructal law provides a methodology that suits this kind of problem well, as it allows us to search for the morphology of a network that facilitates heat and fluid flows. Here, we propose to consider a dendritic architecture. The theoretical discovery of trees stems from the decision to connect a point (source or collector) with an infinity of points (volume, area, line). These tree-shaped geometries have low resistance and facilitate the overall flow [20, 21].

We consider a system with a single inlet and multiple outlets, which would be representative of an underground ventilation distribution network where pre-heated or pre-cooled fresh air would be blown at multiple locations at the same time. In a residential building for example, the air could be blown into the kitchen, dining room and bedrooms simultaneously. At a larger scale, the configuration proposed here may correspond to the supply of pre-heated or pre-cooled air to a new neighborhood with the objective of connecting buildings to the network outlets.

In such a configuration, the challenge is to discover if it is better, from a thermal efficiency point of view, to deliver \dot{Q} through a linear network (as in Section 2) or to consider several outlets thanks to a tree-shaped network, the geometrical features of which (tube diameters and lengths) have to be determined.

3.1 Network features

In section 2, we provided a methodology for designing an EAHE meeting 4 objectives (\dot{Q} , Δh , η and ε). Here, such an EAHE serves as a reference case which has to be improved by re-arranging the single pipe as a tree-shaped network. The first three objectives (\dot{Q} , Δh , η) remain identical for the network, so the mass flow rate is also the same (see Eq. (3)). Finally, this work was achieved under the constraint of constant volume, which is in line with constructal theory.

The bifurcation level N is not fixed but stands as a degree of freedom. The network shape is illustrated in FIGURE 4 when N varies from 1 to 3. Note that for a given network, there are 2^N outlets.

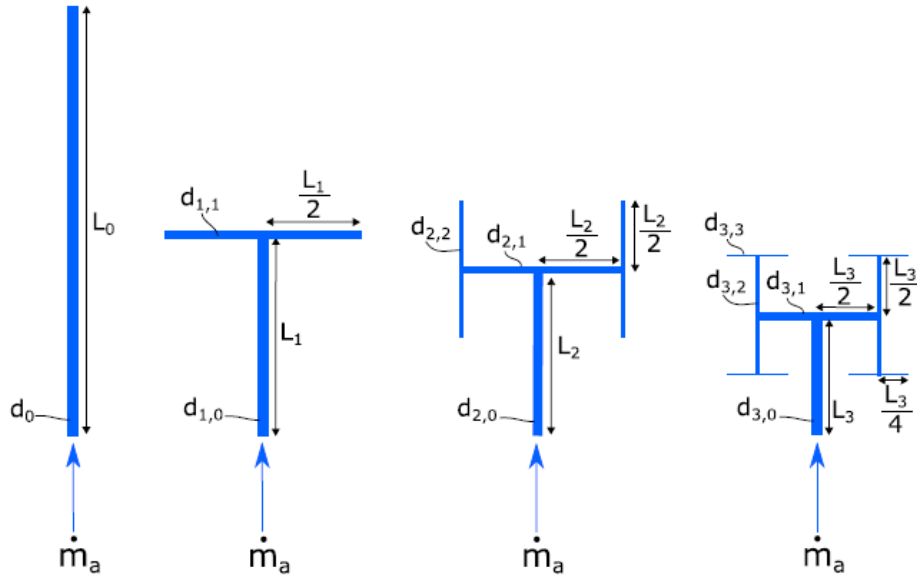


FIGURE 4. Network shape for bifurcation level N ranging from 0 (left, corresponds to the straight pipe) to 3 (right). The overall fluid volume is fixed. The networks are drawn to scale in accordance with the results of Section 3.2.

As mentioned in the previous section, the decrease in enthalpy is already large for a single pipe, which is a consequence of setting a low, yet realistic, value for η . This means that the pipe wall surface is large enough to favor the heat transfer with the ground. This is line with the results provided in Table 3 : a larger EAHE would not increase the cooling power significantly. Therefore improving the network performance means designing an architecture that allows a pressure loss decrease with shorter and wider pipes. For this reason, the length of the first branch of the network is set as a degree of freedom, and the lengths of the other branches are defined as

a fraction of the first, so that the outlets are uniformly spaced. Here, L_N represents the length of the first branch for a network with N bifurcation levels. The diameters of the pipes are allowed to vary from one pairing level to the next, but the overall underground network volume has the same value as a single pipe of diameter d_0 and length L_0 .

The next step is to calculate the enthalpy change and pressure drop between the inlet and outlet of the network according to Eqs. (1) and (9) respectively. Considering that:

- At each bifurcation level, the mass flow rate is equally divided between the 2 new branches;
- The length of a single branch is a fraction of L_N as depicted in FIGURE 4;
- $h_{c,i}$ and $f_{D,i}$ vary from the first branch (subscript 0) to the last (subscript N);
- The diameter $d_{N,i}$ varies depending on the bifurcation level N and on the position i of the branch ($i=0$ being the first branch, just as for $h_{c,i}$ and $f_{D,i}$);

The enthalpy and the pressure drop from entrance to exit are presented in Eqs. (20) and (21), while the network volume is given in Eq. (22). The use of the brackets denotes that the value has been rounded down.

$$\eta = \exp\left(-\frac{\pi L_N}{C_{p,ha} \dot{m}_a} \sum_{i=0}^N h_{c,i} d_{N,i} 2^{\lfloor \frac{i}{2} \rfloor}\right) \quad (20)$$

$$\Delta P = \frac{8 \dot{m}_a^2 L_N}{\rho_{ha} \pi^2} \sum_{i=0}^N \frac{f_{D,i}}{2^{2i + \lfloor \frac{i+1}{2} \rfloor} d_{N,i}^5} \quad (21)$$

$$V = \frac{\pi}{4} L_N \sum_{i=0}^N d_{N,i}^2 2^{\lfloor \frac{i}{2} \rfloor} \quad (22)$$

3.2 Network sizing

Let us consider a set of unknowns — $d_{N,0}, d_{N,1}, \dots, d_{N,N}$ — which has to be determined such that a function — $\Delta P(d_{N,0}, d_{N,1}, \dots, d_{N,N})$ — is minimum under the constraint of constant total volume V . Following the works [22], for heat and fluid flow, and [23], for mass diffusion, the ratio of two consecutive pipe diameters is obtained using the method of Lagrange multipliers [24]. The aggregate function is $\Phi = \Delta P(d_{N,i}) + \lambda V(d_{N,i})$, where λ is the Lagrange multiplier. Its minimum is given by $\partial\Phi/\partial d_{N,i} = 0$. Assuming a constant friction factor the pressure drop is minimum when the diameter ratio, R_d , of two pipes at a bifurcation is:

$$R_d = \frac{d_{N,i+1}}{d_{N,i}} = 2^{-3/7} \quad (23)$$

Note that the tube diameter ratio depends neither on the tube length nor on the mass flow rate. Therefore, it is the same for all the bifurcations and for all the networks.

Thus, the problem reduces to 2 unknowns (L_N and $d_{N,0}$) under the constraint of constant volume V . Additionally, in view of what was described at the beginning of this section, the non-dimensional overall enthalpy was kept at $\eta = 0.05$, Δh was also a constant for a given location, so the cooling power remained the same but the EAHE coefficient of performance ε changed.

Therefore, the set of Eqs. (18-20) was rewritten as:

$$\eta = \exp\left(-\frac{\pi L_N d_{N,0}}{C_{p,ha} \dot{m}_a} \sum_{i=0}^N h_{c,i} 2^{\lfloor \frac{i}{2} \rfloor} R_d^i\right) \quad (24)$$

$$\Delta P = \frac{8 \dot{m}_a^2 L_N}{\rho_{ha} \pi^2 d_{N,0}^5} \sum_{i=0}^N \frac{f_{D,i} R_d^{-5i}}{2^{2i + \lfloor \frac{i+1}{2} \rfloor}} \quad (25)$$

$$V = \frac{\pi}{4} L_N d_{N,0}^2 \sum_{i=0}^N 2^{\lfloor \frac{i}{2} \rfloor} R_d^{2i} \quad (26)$$

Recalling the assumption of a constant volume V , and dividing Eq. (26) by Eq. (24) leads to:

$$d_{N,0} = - \frac{4 V}{\pi \ln(\eta) C_{p,ha} \dot{m}_a} \frac{\sum_{i=0}^N h_{c,i} 2^{\lfloor \frac{i}{2} \rfloor} R_d^i}{\sum_{i=0}^N 2^{\lfloor \frac{i}{2} \rfloor} R_d^{2i}} \quad (27)$$

Next, the length L_N can be obtained from Eq. (26).

$$L_N = \frac{4 V}{\pi d_{N,0}^2 \sum_{i=0}^N 2^{\lfloor \frac{i}{2} \rfloor} R_d^{-2i}} \quad (28)$$

Finally, the pressure drop is computed according to Eq. (25), and the coefficient of performance comes from Eq. (5).

3.3 Comparison between a network and a single pipe

Equations (25), (27) and (28) can be re-arranged in order to highlight the influence of the cooling power and of the enthalpy difference as in Section 2.

$$d_{N,0} = K_{d_N} \left(\frac{\dot{Q}^3}{\Delta h^4} \right)^{\frac{1}{6}} \quad (29)$$

$$L_N = K_{L_N} \left(\frac{\dot{Q}^3}{\Delta h^2} \right)^{\frac{1}{6}} \quad (30)$$

$$\Delta P_N = K_{P_N} \Delta h \quad (31)$$

Next, we define dimensionless numbers that compare the length and diameter of the first branch of the network with those of a single pipe.

$$\tilde{d} = \frac{d_{N,0}}{d_0} = \frac{K_{d_N}}{K_{d_0}} \quad (32)$$

$$\tilde{L} = \frac{L_N}{L_0} = \frac{K_{L_N}}{K_{L_0}} \quad (33)$$

The non-dimensional pressure drop is

$$\widetilde{\Delta P} = \frac{\Delta P_N}{\Delta P_0} \quad (34)$$

From Eqs. (9) and (25), we also have

$$\widetilde{\Delta P} = \frac{L_N}{L_0} \left(\frac{d_0}{d_{N,0}} \right)^5 \frac{\sum_{i=0}^N \frac{f_{D,i} R_d^{-5i}}{2^{2i + \lfloor \frac{i+1}{2} \rfloor}}}{f_{D,0}} \quad (35)$$

Combining Eq. (34) with the definition of the coefficient of performance in Eq. (5), and remembering that the cooling power remains constant, leads to:

$$\tilde{\varepsilon} = \frac{1}{\widetilde{\Delta P}} \quad (36)$$

This latest result shows that, for an expected cooling power, the improvement in thermal efficiency obtained with a network, related to a single pipe, does not depend on the enthalpy difference between the inlet air and the wet pipe wall, which is a way to account for the environmental conditions (climate and soil). This indicates that the design methodology of the dendritic EAHE is robust and can be applied, as a pre-design tool, independently of the EAHE location.

Note that the two coefficients $f_{D,i}$ and $h_{c,i}$ were determined for each branch of the networks by using an iterative procedure, just as in the single pipe case. The results are plotted in FIGURE 5 to 7 for the four cases presented in Table 2.

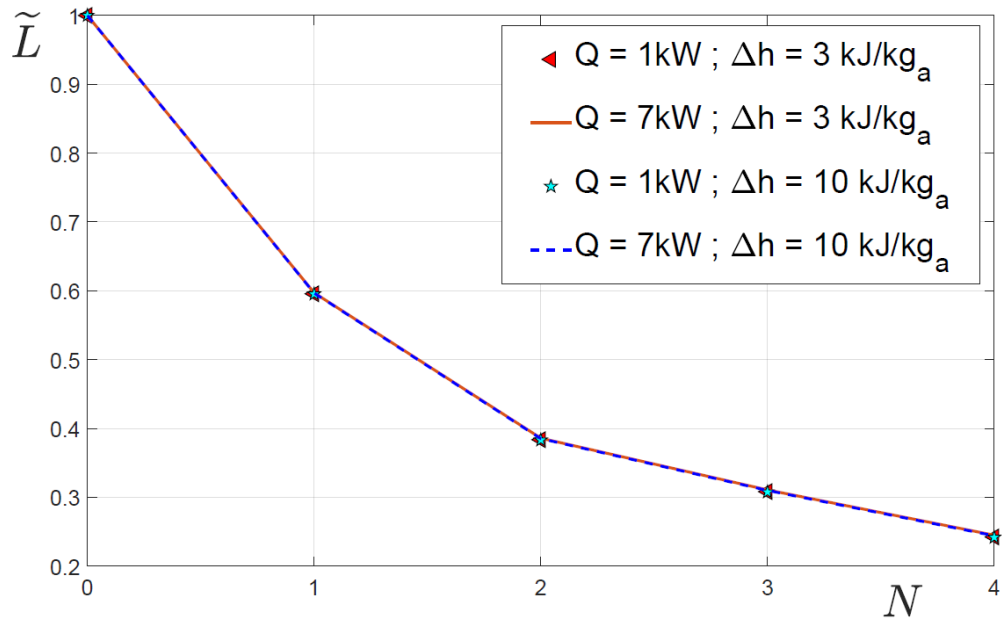


FIGURE 5. Length reduction at the inlet of the network for different bifurcation levels, cooling power and enthalpy difference

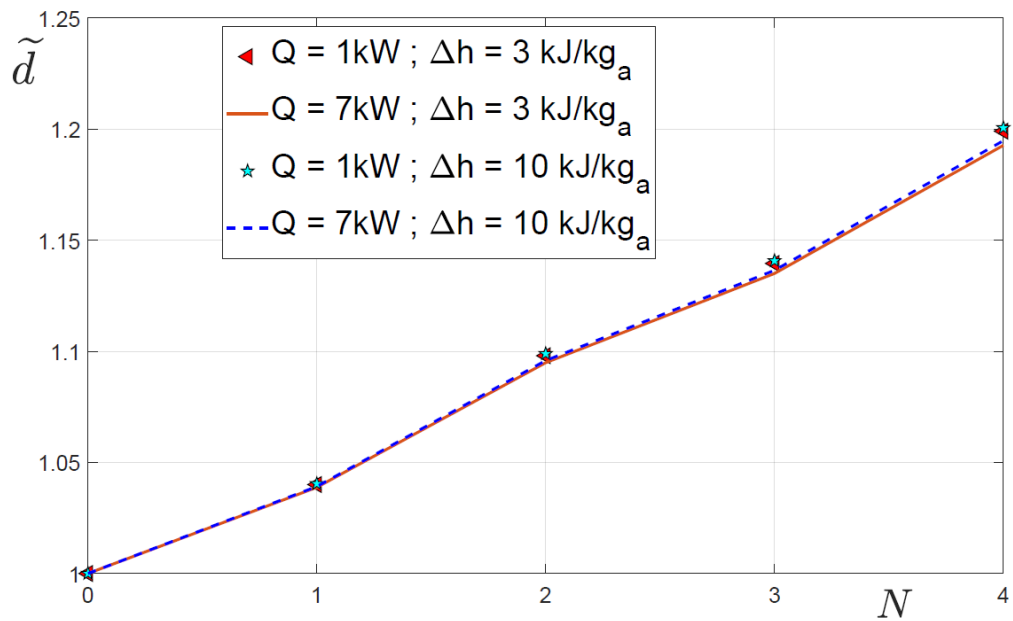


FIGURE 6. Diameter increase at the inlet of the network for different bifurcation levels, cooling power and enthalpy difference

No significant variation is observed for \tilde{d} and \tilde{L} when \dot{Q} and Δh vary. To be more precise, the discrepancy remains lower than $8 \cdot 10^{-3}$ in both cases. This strengthens the result obtained analytically and shows the marginal impact of the friction factor and the convective heat transfer coefficient.

Note that the length of the first branch decreases significantly, becoming 75% shorter for a network when $N = 4$ bifurcations. As a result, the EAHE is much more compact. In the meantime, the diameter at the inlet increases almost linearly with N : it is 20% larger than the single pipe diameter for a bifurcation level of 4.

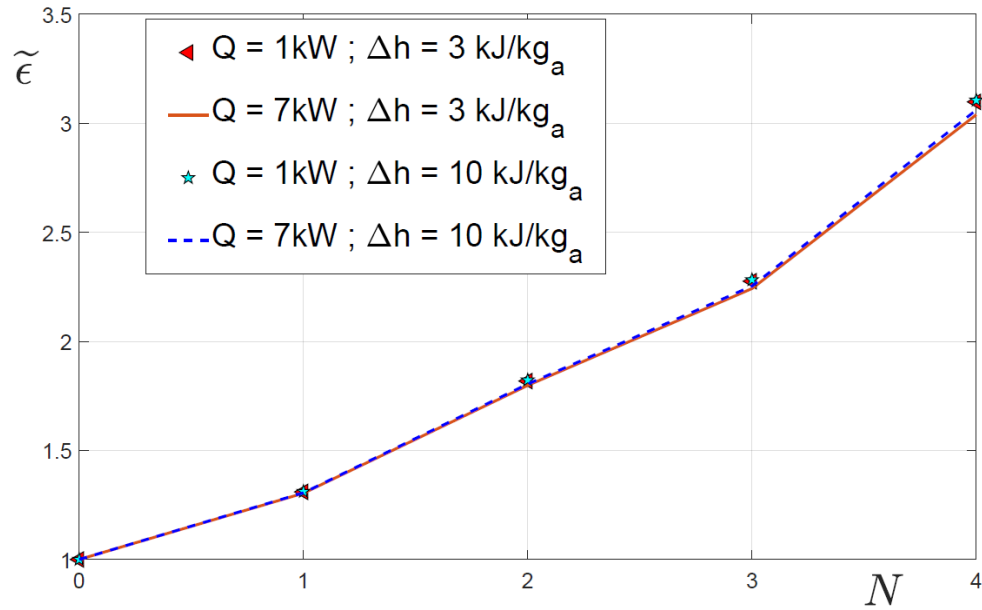


FIGURE 7. Relative increase of the coefficient of performance for different bifurcation levels, cooling power and enthalpy difference

The coefficient of performance increases very significantly with the level of bifurcation, which means that the pressure drop decreases considerably because of the network design, while the cooling power remains constant. $\tilde{\epsilon}$ exceeds 3 for a bifurcation level of 4, meaning that the power

required to blow the air through 16 outlets is one third of that needed for a single outlet, the total mass flow rate being the same.

3.4 Discussion

Even though this work leads to straightforward results, it should not be forgotten that it relies on some simplifying assumptions. First, heat transfers within the ground were not taken into account, the temperature of the latter being considered as a constant. This assumption relies on the practical observation that the ground temperature varies according to climatic excitation, the amplitude of which decreases with depth [25]. As a result, the deeper the EAHE is, the smoother the temperature variations in the ground are. For this reason, the assumption sounds reasonable for a single pipe EAHE. However, arranging the EAHE as a 2D network will lead to undesirable heat transfer close to the bifurcations, the latter behaving like a by-passes. Therefore, there is a risk that a more compact EAHE will exhibit a higher enthalpy at the outlet, which would decrease its coefficient of performance. Second, this study was achieved under the assumption of steady state, whereas the soil behaves as a transient medium. This is a well-known issue in geothermics: the temperature of the ground increases because of heat transfer with the EAHE, simultaneously decreasing its ability to pre-cool the air. The problem can become even more complex if the mass flow rate changes over time.

Therefore, further work is necessary to strengthen the results presented here before considering practical guidelines. For example, the computation of time-dependent heat transfer in the ground would give more accurate boundary conditions for estimating the behavior of the EAHE. In our opinion, this gives us good reason to consider the methodology presented here for pre-design only. Nevertheless, the superiority of the network is clearly demonstrated, which gives confidence in exploring this option in greater details. Moreover, it was demonstrated that this

superiority does not depend on the cooling demand or on the environmental conditions, making this approach suitable for a wide range of applications and climates.

4 CONCLUSION

The main conclusion of this work is that the design of an EAHE should definitely consider the use of a network, as it provides significant energy savings. Independently of the scale of the network, building scale or cluster-of-buildings scale, the methodology developed in the light of the Constructal law argues for more and more compact networks when the available pipes volume is constant. The dendritic network configuration proves to perform better than a classical single pipe design, for a targeted cooling power, considering heat and moisture transfer along the flow path. Major improvements could be found in the search for minimum flow resistances. The approach has proved to be robust as the improved performances of the network do not depend on the environmental conditions (the enthalpy difference between the outdoor air and the soil).

Future work will consider weather data to model the behavior of dendritic networks in dynamic conditions. The cooling/heating demand will also vary accordingly.

5 APPENDIX

Combining Eqs. (7) and (3), we have

$$L_0 = -\frac{C_{p,ha} \ln(\eta)}{\pi h_c} \frac{\dot{Q}}{(1-\eta) \Delta h} \frac{1}{d_0} \quad (37)$$

While the diameter d_0 is given from Eq. (13)

$$d_0 = \left(-\frac{8 f \varepsilon C_{p,ha} \ln(\eta)}{\rho_{ha}^2 \pi^3 (1-\eta)^4 h_c} \right)^{\frac{1}{6}} \left(\frac{\dot{Q}^3}{\Delta h^4} \right)^{\frac{1}{6}} \quad (38)$$

The constant K_{d_0} is expressed by

$$K_{d_0} = \left(-\frac{8 f \varepsilon C_{p,ha} \ln(\eta)}{\rho_{ha}^2 \pi^3 (1-\eta)^4 h_c} \right)^{\frac{1}{6}} \quad (39)$$

Therefore, Eq. (17) becomes

$$L_0 = -\frac{C_{p,ha} \ln(\eta)}{\pi h_c (1-\eta)} \frac{1}{K_{d_0}} \frac{\dot{Q}}{\Delta h} \left(\frac{\dot{Q}^3}{\Delta h^4} \right)^{-\frac{1}{6}} \quad (40)$$

This leads to

$$L_0 = -\frac{C_{p,ha} \ln(\eta)}{\pi h_c (1-\eta) K_{d_0}} \left(\frac{\dot{Q}^3}{\Delta h^2} \right)^{\frac{1}{6}} \quad (41)$$

And the constant K_{L_0} is expressed by

$$K_{L_0} = -\frac{C_{p,ha} \ln(\eta)}{\pi h_c (1-\eta) K_{d_0}} \quad (42)$$

For the pressure drop, we start from Eq. (9), then replace L_0 and d_0 obtained by Eqs. (17) and (18) respectively.

$$\Delta P = \frac{8 f \dot{m}_a^2}{\rho_{ha} \pi^2} K_{L_0} \left(\frac{\dot{Q}^3}{\Delta h^2} \right)^{\frac{1}{6}} \frac{1}{K_{d_0}^5} \left(\frac{\dot{Q}^3}{\Delta h^4} \right)^{-\frac{5}{6}} \quad (43)$$

The mass flow rate is replaced according to Eq. (3).

$$\Delta P = \frac{8 f}{\rho_{ha} \pi^2} \frac{K_{L_0}}{K_{d_0}^5} \left(\frac{\dot{Q}}{(1-\eta) \Delta h} \right)^2 \left(\frac{\dot{Q}^3}{\Delta h^2} \right)^{\frac{1}{6}} \left(\frac{\Delta h^4}{\dot{Q}^3} \right)^{\frac{5}{6}} \quad (44)$$

After some rearrangements, we obtain

$$\Delta P = \frac{8 f}{\rho_{ha} \pi^2 (1-\eta)^2} \frac{K_{L_0}}{K_{d_0}^5} \frac{\dot{Q}^{2+\frac{3}{6}-\frac{15}{6}}}{\Delta h^{2+\frac{2}{6}-\frac{20}{6}}} \quad (45)$$

The constant K_{P_0} is

$$K_{P_0} = \frac{8 f}{\rho_{ha} \pi^2 (1-\eta)^2} \frac{K_{L_0}}{K_{d_0}^5} \quad (46)$$

For the EAHE organized as a network, we start from the definition of the total volume for an EAHE given for a single pipe:

$$V = \frac{\pi}{4} d_0^2 L_0 \quad (47)$$

This equation is then introduced into Eq. (27), where the mass flow rate is replaced by Eq. (3).

$$d_{N,0} = -\frac{1}{\ln(\eta) C_{p,ha}} \frac{\sum_{i=0}^N h_{c,i} 2^{\lfloor \frac{i}{2} \rfloor} R_d^{-i}}{\sum_{i=0}^N 2^{\lfloor \frac{i}{2} \rfloor} R_d^{-2i}} \pi d_0^2 L_0 \frac{(1-\eta) \Delta h}{\dot{Q}} \quad (48)$$

Then, L_0 and d_0 are replaced by Eqs. (17) and (18) respectively.

$$d_{N,0} = -\frac{\pi}{\ln(\eta) C_{p,ha}} \frac{\sum_{i=0}^N h_{c,i} 2^{\lfloor \frac{i}{2} \rfloor} R_d^{-i}}{\sum_{i=0}^N 2^{\lfloor \frac{i}{2} \rfloor} R_d^{-2i}} \frac{(1-\eta) \Delta h}{\dot{Q}} \left(K_{d_0} \left(\frac{\dot{Q}^3}{\Delta h^4} \right)^{\frac{1}{6}} \right)^2 K_{L_0} \left(\frac{\dot{Q}^3}{\Delta h^2} \right)^{\frac{1}{6}} \quad (49)$$

The terms are rearranged as follows:

$$d_{N,0} = -\frac{\pi (1-\eta) K_{d_0}^2 K_{L_0} \sum_{i=0}^N h_{c,i} 2^{\lfloor \frac{i}{2} \rfloor} R_d^{-i}}{\ln(\eta) C_{p,ha} \sum_{i=0}^N 2^{\lfloor \frac{i}{2} \rfloor} R_d^{-2i}} \frac{\dot{Q}^{-1+\frac{6}{6}+\frac{1}{6}}}{\Delta h^{\frac{8}{6}+\frac{2}{6}-1}} \quad (50)$$

The constant K_{d_N} is finally obtained.

$$K_{d_N} = -\frac{\pi (1-\eta) K_{d_0}^2 K_{L_0} \sum_{i=0}^N h_{c,i} 2^{\lfloor \frac{i}{2} \rfloor} R_d^{-i}}{\ln(\eta) C_{p,ha} \sum_{i=0}^N 2^{\lfloor \frac{i}{2} \rfloor} R_d^{-2i}} \quad (51)$$

Next, we look at the length of the first branch in the network, defined in Eq. (28). Combined with Eq. (29), it gives:

$$L_N = -\frac{1}{d_{N,0}^2 \sum_{i=0}^N 2^{\lfloor \frac{i}{2} \rfloor} R_d^{-2i}} d_0^2 L_0 \quad (52)$$

Finally,

$$L_N = -\frac{1}{\sum_{i=0}^N 2^{\lfloor \frac{i}{2} \rfloor} R_d^{-2i}} \left(\frac{K_{d_0}}{K_{d_N}} \right)^2 K_{L_0} \left(\frac{\dot{Q}^3}{\Delta h^2} \right)^{\frac{1}{6}} \quad (53)$$

The constant K_{L_N} is:

$$K_{L_N} = -\frac{1}{\sum_{i=0}^N 2^{\lfloor \frac{i}{2} \rfloor} R_d^{-2i}} \left(\frac{K_{d_0}}{K_{d_N}} \right)^2 K_{L_0} \quad (54)$$

The last equation to be rearranged is Eq. (25) which defines the pressure losses in the network. Combined with Eqs. (32) and (33), it leads to:

$$\Delta P = \frac{8 \dot{m}_a^2}{\rho_{ha} \pi^2} \sum_{i=0}^N \frac{f_i R_d^{5i}}{2^{2i + \lfloor \frac{i+1}{2} \rfloor}} K_{LN} \left(\frac{\dot{Q}^3}{\Delta h^2} \right)^{\frac{1}{6}} \left(K_{LN} \left(\frac{\dot{Q}^3}{\Delta h^2} \right)^{\frac{1}{6}} \right)^{-5} \quad (55)$$

This expression is rearranged as follows:

$$\Delta P = \frac{8 \dot{m}_a^2}{\rho_{ha} \pi^2} \sum_{i=0}^N \frac{f_i R_d^{5i}}{2^{2i + \lfloor \frac{i+1}{2} \rfloor}} \frac{K_{LN}}{K_{dN}^5} \frac{\dot{Q}^{-2}}{\Delta h^{-3}} \quad (56)$$

Next, the mass flow rate is replaced by Eq. (3).

$$\Delta P = \frac{8}{\rho_{ha} \pi^2} \frac{K_{LN}}{K_{dN}^5} \sum_{i=0}^N \frac{f_i R_d^{5i}}{2^{2i + \lfloor \frac{i+1}{2} \rfloor}} \frac{\dot{Q}^{-2}}{\Delta h^{-3}} \left(\frac{\dot{Q}}{(1-\eta) \Delta h} \right)^2 \quad (57)$$

This leads to:

$$\Delta P = \frac{8}{\rho_{ha} \pi^2 (1-\eta)^2} \frac{K_{LN}}{K_{dN}^5} \sum_{i=0}^N \frac{f_i R_d^{5i}}{2^{2i + \lfloor \frac{i+1}{2} \rfloor}} \Delta h \quad (58)$$

Finally, the constant K_{PN} is:

$$K_{PN} = \frac{8}{\rho_{ha} \pi^2 (1-\eta)^2} \frac{K_{LN}}{K_{dN}^5} \sum_{i=0}^N \frac{f_i R_d^{5i}}{2^{2i + \lfloor \frac{i+1}{2} \rfloor}} \quad (59)$$

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