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Variational asymptotic micromechanics modeling of continuous carbon fibers and radially aligned carbon nanotubes reinforced composite materials

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Abstract

This study presents a micromechanics model based on the variational asymptotic method for unit cell homogenization (VAMUCH) to predict the effective elastic properties of a continuous fuzzy fiber-reinforced composite material. The constructional feature of the fuzzy fiber-reinforced composite (FFRC) proposed in [S.I. Kundalwal, M.C. Ray, “Effective properties of a novel continuous fuzzy-fiber reinforced composite using the method of cells and the finite element method”, *European Journal of Mechanics A/Solids* 36 (2012) 191-203] is that uniformly aligned carbon nanotubes (CNTs) are radially grown on the surface of the horizontal carbon fibers. The effective elastic properties of the fuzzy fiber-reinforced composite estimated by the variational asymptotic method have been compared with those predicted by the finite element based micromechanics approach (FEA) and the method of cells (MOC) approach reported in the previous reference. It has been also observed that the transverse effective properties of such a composite material are significantly improved due to the radial growing of CNTs on the surface of the carbon fiber. It is shown that two 2D unit cells can be successfully used within the VAMUCH for the intuitive prediction of the full effective 3D anisotropic properties of such FFRC. This lead to a reduced computational expenditure in comparison to the three 3D unit cells used in the FEA reported in the above reference.

Keywords: Nano-structures, Mechanical properties, Micro-mechanics

1. Introduction

Due to their exceptional mechanical properties, carbon nanotubes (CNTs) have been extensively used as reinforcements in composite materials. Since the last decade, researches related to the nanotubes and their co-related composite materials have been dramatically increased. Critical reviews on nanotubes and nanotubes/nanoclays related polymer composite materials can be found for example in [1, 2]. The attractive elastic properties of CNTs continue to encourage researchers to develop novel CNT-reinforced

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polymer matrix composites. Very recently, Kundalwal and Ray [3, 4] have proposed novel short and continuous fuzzy fiber-reinforced composite materials. The proposed composite materials are composed of single-walled zig-zag CNTs, carbon fibers and a polymer matrix. The carbon fiber reinforcements are horizontally aligned and CNTs are radially grown on the surfaces of these carbon fibers. A micromechanics model based on the method of cells has been used to predict the effective elastic properties of a lamina made of the continuous fuzzy fiber-reinforced composite while a micromechanics model based on the Mori-Tanaka method has been used to predict the effective elastic properties of a lamina made of the short fuzzy fiber-reinforced composite. It has been shown that the transverse properties of the proposed fuzzy fiber-reinforced composite are significantly improved without altering the values of the in-plane effective elastic constants.

Nowadays, it is well recognized that, for this type of composite materials, the traditional continuum mechanics is no longer suitable because the dimensions of the CNTs are within nanoscale while the polymer itself is often regarded as a bulk matrix material. The mechanical properties of CNT-reinforced nanocomposites have been modeled by many researchers using the molecular dynamics (MD) simulation, continuum mechanics, and multi-scale simulation, see [3–6] and references cited therein. The continuum mechanics modeling approaches have recently received more attention because of their reduced computational expenditure. Numerous continuum mechanics modeling approaches have been proposed to predict the effective properties of composite materials from known constituents informations (e.g., properties, topology and morphology textures). Roughly, the micromechanics models for composite materials can be classified in two categories: (a) mean field (MF)-based models and (b) local field (LF)-based models. The MF-based methods (see [7]) are based on Eshelby’s solutions [8] and are capable of predicting the entire behavior of composite materials under arbitrary loads. They can provide reasonable estimates for a material’s bulk elastic response but typically fail to provide good estimates for the local responses and the history-dependent responses of the material. They use averaged representations of the local fields within the constituents of the composite, i.e., they do not account for the local fluctuations of the field quantities. In order to correctly predict the local and bulk response characteristics in the elastic and inelastic domains, it is necessary to utilize micromechanical theories that consider both the average fields within phases as well as the fluctuating fields within the phases. LF-based theories consider both the average fields within phases as well as the fluctuating fields within the phases. There are a number of such theories currently available (see [9] and references cited therein).

Here in this paper, we will provide a micromechanics modeling of continuous fuzzy-fiber reinforced nanocomposites based on the Variational Asymptotic Method for Unit Cell Homogenization (VA-

MUCH), a recently developed micromechanics modeling framework [10–12]. This method is based on the variational asymptotic method (VAM) [13, 14] which is applicable to any solid mechanics problem admitting a variational structure where one or more relatively small parameters are involved. The “smallness” of these parameters is exploited by using an asymptotic expansion structure of the functional of the problem (and not of the unknown field quantities as done in the traditional asymptotic methods). Thus, VAM combines the advantages of both variational (most notably finite element (FE) structure) and asymptotic methods. As pointed out in [10–12], in comparison to existing FE-based micromechanics approaches, VAMUCH has the following features:

- It adopts the variational asymptotic method as its mathematical foundation. It has the same rigor as mathematical homogenization theories (MHT) without even assuming periodic fluctuation functions and boundary conditions. It uses only assumptions inherent in micromechanics without invoking any additional *ad hoc* assumptions.
- It has an inherent variational nature and its numerical implementation is shown to be straightforward.
- It handles 1D/2D/3D unit cells uniformly. The dimensionality of the problem is determined by that of the periodicity of the unit cell.
- It can obtain different material properties in different directions simultaneously, which is more efficient than those approaches requiring multiple runs under different loading conditions.
- It calculates effective properties and local fields directly with the same accuracy as the fluctuation functions. No post-processing calculations such as stress averaging and strain averaging are needed.

Although VAMUCH is as versatile as FE-based approaches, VAMUCH is dramatically different from FEM-based approaches, both from the view point of theoreticians and from the view point of practicing engineers. The present work aims at the intuitive prediction of the effective properties of the FFRC by VAMUCH using only two two-dimensions (2D) unit-cells (UCs) instead of three three-dimensions (3D) UCs used in the FEA reported in [4]. Note that, all results presented herein are obtained using GetFem++ [15] as a FE library and Gmsh [16] as a meshing tool.

2. Effective elastic properties of the fuzzy fiber-reinforced composite

Figure 1 shows a schematic diagram of the concept of continuous fuzzy-fiber reinforced composite proposed in [4]. This continuous unidirectional carbon fiber-reinforced composite is constructed in a way

that CNTs of equal length are uniformly aligned and radially grown on the surface of the carbon fiber reinforcements. As in [4, 5], CNTs are considered here to be transversely isotropic. By embedding such a fuzzy-fiber into the polymer material, the gap between the CNTs is filled with the polymer. Therefore, the radially aligned CNTs reinforce the polymer matrix surrounding the carbon fiber along the transverse direction of the carbon fiber. The representative volume element (RVE) of the fuzzy fiber-reinforced composite (FFRC) is treated herein as being composed of three phases wherein the reinforcement is the carbon fiber coated by a polymer matrix nanocomposite (PMNC) and the matrix is the monolithic polymer material, see Fig. 2. Consequently the micromechanics model for estimating the effective elastic properties of this FFRC starts with the estimation of the effective elastic properties of the PMNC. In what follows, the VAMUCH micromechanics modeling approach is presented and applied to compute the effective elastic properties of the PMNC and FFRC.

2.1. Variational asymptotic method for periodically heterogeneous materials

The general VAMUCH theory in [10] can be specified for micromechanical analysis of continuous fiber-reinforced composites as the minimization of the following functional

$$\Pi = \frac{1}{2} \left\langle C_{ijkl} \left[\bar{\epsilon}_{ij} + \frac{1}{2} (\chi_{i,j} + \chi_{j,i}) \right] \left[\bar{\epsilon}_{kl} + \frac{1}{2} (\chi_{k,l} + \chi_{l,k}) \right] \right\rangle, \quad (1)$$

subject to the following constraints

$$\chi_i \left(y_1, y_2 = -\frac{d_2}{2}, y_3 \right) = \chi_i \left(y_1, y_2 = +\frac{d_2}{2}, y_3 \right), \quad (2)$$

$$\chi_i \left(y_1, y_2, y_3 = -\frac{d_3}{2} \right) = \chi_i \left(y_1, y_2, y_3 = +\frac{d_3}{2} \right), \quad (3)$$

$$\langle \chi_i \rangle = 0. \quad (4)$$

Here the angle brackets denote averaging over the UC, C_{ijkl} the positionally dependent fourth-order elasticity tensor, $\bar{\epsilon}_{ij}$ the macroscopic strain field, χ_i the fluctuating functions, $\chi_{i,j} = \partial \chi_i / \partial y_j$, y_i the Cartesian coordinates describing the UC with y_2 and y_3 originated from the middle of the UC, and d_2 and d_3 the UC lengths in y_2 and y_3 directions respectively. Throughout this paper, Latin indices assume 1, 2, and 3 and repeated indices are summed over their range except where explicitly indicated. Equations (2) and (3) express the traditional periodic boundary conditions at $(y_2 = -d_2/2, y_2 = +d_2/2)$ and $(y_3 = -d_3/2, y_3 = +d_3/2)$ respectively. As it is shown in [10], Eq. (4) does not affect the minimum value of the functional in Eq. (1) but helps uniquely to determine the fluctuating functions, χ_i . Here it is assumed that the continuous fiber axis is parallel to the y_1 -axis. Therefore, the partial derivatives of the fluctuating functions, $\chi_{i,1}$, will vanish because the continuous fiber-reinforced composite is uniform in the y_1 direction.

The functional (1) can be restated in a matrix form as minimizing the following functional

$$\Pi = \frac{1}{2\Omega} \int_{\Omega} \{\bar{\varepsilon} + \varepsilon\}^{\top} [\mathbf{D}] \{\bar{\varepsilon} + \varepsilon\} d\Omega, \quad (5)$$

where

$$\varepsilon = \{0 \quad \chi_{2,2} \quad \chi_{3,3} \quad \chi_{2,3} + \chi_{3,2} \quad \chi_{1,3} \quad \chi_{1,2}\}^{\top} = \mathcal{H} \{\chi_1 \quad \chi_2 \quad \chi_3\}^{\top} \equiv \mathcal{H} \chi. \quad (6)$$

$[\mathbf{D}]$ is the 6×6 material matrix condensed from the positionally dependent fourth-order elasticity tensor, C_{ijkl} , Ω is the area of the UC, \mathcal{H} is a 6×3 matrix of differential operators defined as

$$\mathcal{H} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & \frac{\partial}{\partial y_2} & 0 \\ 0 & 0 & \frac{\partial}{\partial y_3} \\ 0 & \frac{\partial}{\partial y_3} & \frac{\partial}{\partial y_2} \\ \frac{\partial}{\partial y_3} & 0 & 0 \\ \frac{\partial}{\partial y_2} & 0 & 0 \end{bmatrix}. \quad (7)$$

If one discretizes χ using the finite elements as

$$\chi(x_i; y_i) = S(y_2, y_3) \Gamma(x_i), \quad (8)$$

where $S(y_2, y_3)$ representing the shape functions (in the assembled sense) and Γ a column matrix of the nodal values of the fluctuation functions (considering the constraints (2) and (3)), one obtains a discretized version of the functional as

$$\Pi = \frac{1}{2\Omega} \left(\Gamma^{\top} \mathbf{D}_{\chi\chi} \Gamma + 2\Gamma^{\top} \mathbf{D}_{\chi\varepsilon} \bar{\varepsilon} + \bar{\varepsilon}^{\top} \mathbf{D}_{\varepsilon\varepsilon} \bar{\varepsilon} \right), \quad (9)$$

where

$$\mathbf{D}_{\chi\chi} = \int_{\Omega} [\mathcal{H} S]^{\top} [\mathbf{D}] [\mathcal{H} S] d\Omega, \quad (10)$$

$$\mathbf{D}_{\chi\varepsilon} = \int_{\Omega} [\mathcal{H} S]^{\top} [\mathbf{D}] d\Omega, \quad (11)$$

$$\mathbf{D}_{\varepsilon\varepsilon} = \int_{\Omega} [\mathbf{D}] d\Omega. \quad (12)$$

In Eq. (8), x_i are used as the global coordinates to describe the macroscopic structure and y_i as the local coordinates to describe the UC. The minimization of Eq. (9) gives the following linear system

$$\mathbf{D}_{\chi\chi} \Gamma = -\mathbf{D}_{\chi\varepsilon} \bar{\varepsilon}. \quad (13)$$

The solution of Eq. (13) can be written as

$$\Gamma = \Gamma_0 \bar{\varepsilon}, \quad (14)$$

where Γ_0 is the solution of the linear system

$$\mathbf{D}_{\chi\chi}\Gamma_0 = -\mathbf{D}_{\chi\varepsilon}. \quad (15)$$

Here, the linear system (15) is solved using the multifrontal massively parallel sparse direct solver (MUMPS), see [17, 18]. Substituting Eq. (14) into Eq. (9), one can calculate the energy stored in the UC as

$$\Pi = \frac{1}{2\Omega} \bar{\varepsilon}^\top \left(\Gamma_0^\top \mathbf{D}_{\chi\varepsilon} + \mathbf{D}_{\varepsilon\varepsilon} \right) \bar{\varepsilon} \equiv \frac{1}{2} \bar{\varepsilon}^\top \bar{\mathbf{D}} \bar{\varepsilon}, \quad (16)$$

where $\bar{\mathbf{D}}$ is the effective (or homogenized) 6×6 material matrix.

Having obtained the effective elastic properties, one can use these properties to carry out the macroscopic analysis of the complete structure to predict the global elastic behavior of the engineering system made of the FFRC. If one needs the point-wise distribution of the elastic fields (displacements, strains, stresses) within the microstructure, one has to uniquely determine the fluctuation functions, χ_i , using the constraints in Eq. (4) and follow the recovery procedure explained in the works related to the VAMUCH, see [19] for more details. Although it is easy to distinguish the VAMUCH from other analytical micromechanics approaches, the VAMUCH is often confused as one of the finite element analysis (FEA)-based micromechanics approaches because the equations of the VAMUCH theory are solved using the FE technique. FEA-based micromechanics approaches carry out a conventional FEA of a UC with specially designed boundary conditions under specifically designed loads.

2.2. Application to continuous fuzzy-fiber reinforced composite material

This section presents the application of the VAMUCH to predict the effective elastic properties of the polymer matrix nanocomposite material surrounding the carbon fiber and the fuzzy fiber-reinforced composite, respectively.

2.2.1. Effective properties of the polymer matrix nanocomposite

As pointed out in [5], the estimation of the effective properties of the polymer matrix nanocomposite material surrounding the carbon fiber requires the consideration of the non-bonded van der Waals interaction between an atom of a CNT and an atom of the polymer, which fills the gap between CNTs. An interphase is considered between a CNT and the polymer matrix which characterizes the non-bonded van

der Waals interaction between this CNT and the polymer matrix [4, 5]. This interphase is considered as an equivalent continuum solid. The effective elastic properties of such an interphase determined by the MD simulation are available in the open literature, see [4, 5]. The effective elastic properties of such an interphase reported in [4] will be used here.

The constructional feature of the FFRC, see Fig. 1, can be viewed as concentric cylinders in which the carbon fiber is wrapped by a lamina of the PMNC, see Fig. 3. This unwound lamina of the PMNC is reinforced by nano-fibers (CNT + Interphase) along its thickness direction (3-direction) as shown in Fig. 3. The effective elastic properties, $\bar{\mathbf{D}}^{\text{LM}}$, of such unwound lamina of the PMNC can be estimated using 2D UC (see Fig. 2b) with the VAMUCH presented in Section 2.1. As stressed out in [4], $\bar{\mathbf{D}}^{\text{LM}}$ directly provides the effective elastic properties at a point in the portion of the PMNC surrounding the carbon fiber where the nano-fiber is aligned with the 3-axis. However, for the point located in the PMNC where the nano-fiber is oriented at an angle, θ , with the 3-axis in the 2-3 plane, $\bar{\mathbf{D}}^{\text{LM}}$ also provides the effective elastic properties with respect to the local (1', 2', 3') material coordinate system. Therefore, the location dependent effective elastic coefficient matrix, $\bar{\mathbf{D}}^{\text{LPMNC}}$, at any point of the PMNC with respect to the 1-2-3 coordinate system can be obtained by the classical transformation rule

$$\bar{\mathbf{D}}^{\text{LPMNC}} = \mathbf{T}^{-\top} \bar{\mathbf{D}}^{\text{LM}} \mathbf{T}^{-1}, \quad (17)$$

where

$$\mathbf{T} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & c^2 & s^2 & cs & 0 & 0 \\ 0 & s^2 & c^2 & -cs & 0 & 0 \\ 0 & -2cs & 2cs & c^2 - s^2 & 0 & 0 \\ 0 & 0 & 0 & 0 & c & -s \\ 0 & 0 & 0 & 0 & s & c \end{bmatrix}, \quad (18)$$

with $c = \cos(\theta)$ and $s = \sin(\theta)$.

Without loss of generality, the volume average of $\bar{\mathbf{D}}^{\text{LPMNC}}$ over the volume of the PMNC can be treated as the constant effective elastic properties, $\bar{\mathbf{D}}^{\text{PMNC}}$, of the PMNC surrounding the carbon fiber with respect to the 1-2-3 coordinate axes of the FFRC. $\bar{\mathbf{D}}^{\text{PMNC}}$ is defined as [4]

$$\bar{\mathbf{D}}^{\text{PMNC}} = \frac{1}{2\pi} \int_0^{2\pi} \bar{\mathbf{D}}^{\text{LPMNC}} d\theta. \quad (19)$$

The procedure to estimate the effective elastic properties, $\bar{\mathbf{D}}^{\text{PMNC}}$, of the PMNC can be summarized as

- use the 2D UC (see Fig. 2b) with the VAMUCH using the elastic properties of the CNT, the interphase and the monolithic polymer to compute $\bar{\mathbf{D}}^{\text{LM}}$;

- use the expressions (17) and (19) to compute $\bar{\mathbf{D}}^{\text{PMNC}}$.

2.2.2. Effective properties of the fuzzy fiber-reinforced composite

Having obtained the effective elastic properties, $\bar{\mathbf{D}}^{\text{PMNC}}$, of the PMNC (see Section 2.2.1) one can use these properties to compute the effective elastic properties, $\bar{\mathbf{D}}^{\text{FFRC}}$, of the FFRC. To this end, the VAMUCH is again used with the 2D UC (see Fig. 2a) and the properties, $\bar{\mathbf{D}}^{\text{PMNC}}$, of the PMNC and those of the carbon fiber and the monolithic polymer. Thus, with the VAMUCH two 2D UCs can be used to obtain the complete 3D effective anisotropic elastic properties of the continuous fuzzy fiber-reinforced composite materials proposed in [4].

3. Results and discussion

In this section, some numerical values of the effective elastic properties of the FFRC are evaluated using the two 2D UCs with the VAMUCH. The material properties of the zig-zag (10, 0) single-walled CNTs, interphase, carbon fiber and polymer considered for evaluating the numerical results are listed in Table 1. The value of the diameter of the carbon fiber is $2a = 10 \mu\text{m}$. The maximum volume fraction of CNTs (V_{CNT}) in the FFRC depends on the CNT diameter, the carbon fiber diameter and the surface to surface distance of two adjacent radially aligned CNTs at their roots [4]. Since in the PMNC, the polymer molecules fill the gap between CNTs to create an interphase, the surface to surface distance between the two adjacent CNTs at their roots is considered as 1.7 nm. The maximum value of V_{CNT} is determined as (see [4] for more details)

$$V_{\text{CNT}}^{\text{max}} = \frac{\pi d_n^2}{2(d_n + 1.7)^2} \left(\sqrt{\frac{\pi v_{\text{CF}}}{2\sqrt{3}}} - v_{\text{CF}} \right), \quad (20)$$

where d_n is the diameter of the CNT, v_{CF} the carbon fiber volume fraction in the FFRC. In this study, $d_n = 0.78 \text{ nm}$. The maximum volume fraction, $v_{\text{NT}}^{\text{max}}$, of the CNTs with respect to the volume of the PMNC (see Fig. 2b) is given as (see [4, Appendix A 8])

$$v_{\text{NT}}^{\text{max}} = \frac{2\sqrt{3}D^2}{\pi(D^2 - d^2)} V_{\text{CNT}}^{\text{max}}, \quad (21)$$

where $d = 2a$, D is the diameter of the carbon fiber and its coating in the FFRC (see Fig. 2a). The volume fraction of the interphase in the PMNC is determined using the gap filled by the polymer molecules between CNTs to create the interphase. Here the value, $\text{Gap} = 0.3333 \text{ nm}$, is used as reported in [4]. For all the results presented in the paper, the volume fraction, v_{FF} , of the fuzzy fiber (carbon fiber and its coating) in the FFRC is as taken as 0.90 instead of 0.9069 used in [4]. Thus for a given value of v_{CF} , the effective properties, $\bar{\mathbf{D}}^{\text{PMNC}}$, of the PMNC is computed using the procedure presented in Section 2.2.1

with the 2D UC for the PMNC (see Fig. 2b). Then $\bar{\mathbf{D}}^{\text{PMNC}}$ is used to compute the effective properties, $\bar{\mathbf{D}}^{\text{FFRC}}$, of the FFRC with the 2D UC for the FFRC (see Fig. 2a). In what follows, unless otherwise mentioned, the effective properties of the FFRC are computed for a particular value of v_{CF} while the maximum value of V_{CNT} corresponding to this value of v_{CF} is used for computing the effective elastic properties. In Figs. 4 to 8, MOC refers to the method of cells while FEA refers to the finite element analysis. These results reported in [4] are compared to those obtained by the VAMUCH presented in the study.

Figure 4 presents the variation of the effective elastic coefficient, $\bar{D}_{11}^{\text{FFRC}}$, of the FFRC with the carbon fiber volume fraction, v_{CF} . All the models predict almost identical estimates for $\bar{D}_{11}^{\text{FFRC}}$. It is seen that the radial growing of aligned CNTs on the surface of the carbon fibers influences negligibly the magnitude of the effective elastic coefficient, $\bar{D}_{11}^{\text{FFRC}}$, when compared with that of the elastic coefficient, $\bar{D}_{11}^{\text{FFRC}}$, without CNTs (i.e. $V_{\text{CNT}} = 0$). This can be explained from Table 1 where one can see that the elastic properties of the CNT transverse to its axis (i.e., in 1-direction) are much less than those along its axis (i.e., in 3-direction). Fig. 5 presents the variation of the effective elastic coefficient, $\bar{D}_{22}^{\text{FFRC}}$, of the FFRC with the carbon fiber volume fraction, v_{CF} . For $V_{\text{CNT}} \neq 0$, all the models yield almost similar results for the value of $\bar{D}_{22}^{\text{FFRC}}$. Contrary to the effective elastic coefficient, $\bar{D}_{11}^{\text{FFRC}}$, it is observed that the radially grown CNTs significantly enhance the out-of-plane effective elastic coefficient, $\bar{D}_{22}^{\text{FFRC}}$, when compared to $\bar{D}_{22}^{\text{FFRC}}$ without CNTs. This is due to the fact that the radially grown CNTs improve the out-of-plane stiffness of the PMNC surrounding the carbon fibers. Since the fuzzy fiber-reinforced composite is transversely isotropic, similar results are also obtained for the effective elastic coefficient, $\bar{D}_{33}^{\text{FFRC}}$. Fig. 6 shows the same trend for the effective elastic coefficient, $\bar{D}_{23}^{\text{FFRC}}$ where a very small discrepancy is observed between the models. The result obtained by VAMUCH is more close to that obtained by MOC. One can observe from Fig. 7 that the growing of the CNTs on the surface of the carbon fiber slightly enhances the value of the effective elastic coefficient, $\bar{D}_{12}^{\text{FFRC}}$, of the FFRC. All the models predict almost similar results for the value of $\bar{D}_{12}^{\text{FFRC}}$ when $V_{\text{CNT}} \neq 0$. Similar results are also obtained for the effective elastic coefficient, $\bar{D}_{13}^{\text{FFRC}}$. Finally, Fig. 8 shows the variation of the effective elastic coefficient, $\bar{D}_{55}^{\text{FFRC}}$, of the FFRC with the carbon fiber volume fraction, v_{CF} . The radially grown CNTs on the surface of the carbon fiber does not enhance significantly $\bar{D}_{55}^{\text{FFRC}}$ when compared with the effective elastic coefficients, $\bar{D}_{22}^{\text{FFRC}}$, $\bar{D}_{23}^{\text{FFRC}}$ and $\bar{D}_{12}^{\text{FFRC}}$.

4. Conclusion

An intuitive prediction of the full effective anisotropic properties of the continuous fuzzy fiber-reinforced composite proposed in [4] using the VAMUCH has been presented. The continuous fuzzy fiber-reinforced composite is considered here as a three-phase composite material where the Carbon fiber coated by a polymer matrix nanocomposite reinforces the monolithic polymer. The polymer matrix nanocomposite is also considered as a three-phase composite material where the CNT coated by an interphase (due to the non-bonded van der Waals interaction between an atom of a CNT and an atom of the polymer) reinforces the monolithic polymer. The VAMUCH is applied to two 2D unit cells to predict the full effective 3D anisotropic properties of the continuous fuzzy fiber-reinforced composite. The VAMUCH can obtain the complete set of material effective physical properties within one analysis without applying any load and any boundary conditions, which is far more efficient and less labor intensive than those approaches requiring multiple runs under different boundary and load conditions. The effective elastic constants of the fuzzy fiber-reinforced composite predicted by the VAMUCH are found to be in a good agreement with those predicted by the FE model using three 3D unit cells and the method of cells model using four 3D unit cells (1 UC for the nano-Fiber i.e. CNT with its interphase, 1 for the PMNC i.e. nano-Fiber and the polymer, 1 UC for the composite fuzzy-fiber i.e. the carbon fiber and the PMNC and finally 1 UC for the FFRC i.e. the composite fuzzy-fiber and the polymer) reported in [4]. As pointed out in [4], because the CNTs are radially grown on the surface of the carbon fiber with its axis normal to this surface, the transverse effective properties, $\overline{D}_{22}^{\text{FFRC}}$, $\overline{D}_{33}^{\text{FFRC}}$, $\overline{D}_{23}^{\text{FFRC}}$, $\overline{D}_{12}^{\text{FFRC}}$, $\overline{D}_{13}^{\text{FFRC}}$ and $\overline{D}_{55}^{\text{FFRC}}$, of the fuzzy fiber-reinforced composite are significantly improved over their values without CNTs. Since the effective elastic constants of the fuzzy fiber-reinforced composite predicted by the VAMUCH are in a good agreement with those reported in [4], the VAMUCH may be efficiently used for intuitive predictions of the effective elastic properties of such continuous fuzzy fiber-reinforced composites and if need be, the VAMUCH can be also used to recovery the local fields within such FFRC due to applied macroscopic loads.

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Figures

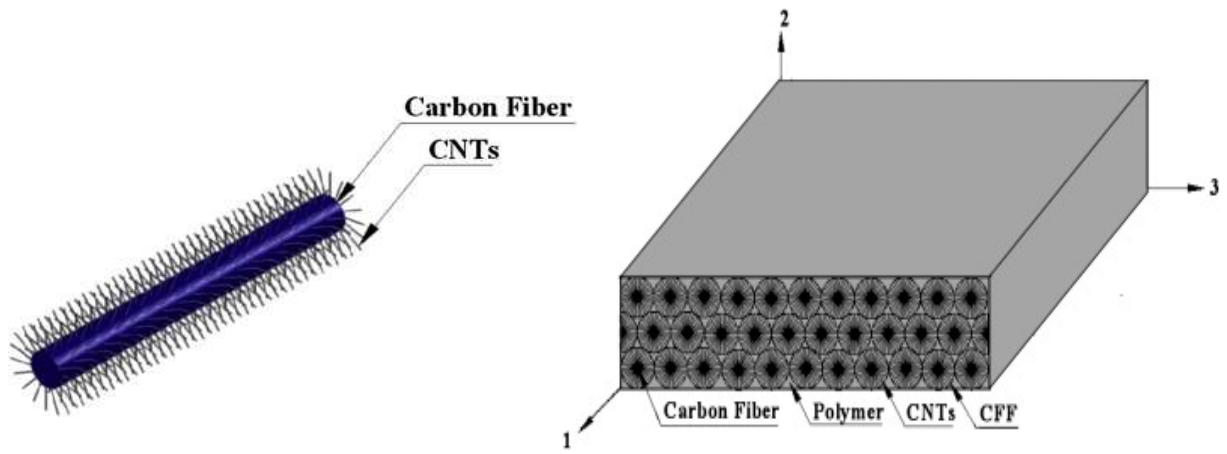


Figure 1: Fuzzy fiber with CNTs radially grown on its surface and a lamina of the fuzzy fiber-reinforced composite containing composite fuzzy fiber (CFF) and polymer matrix. Taken from [4].

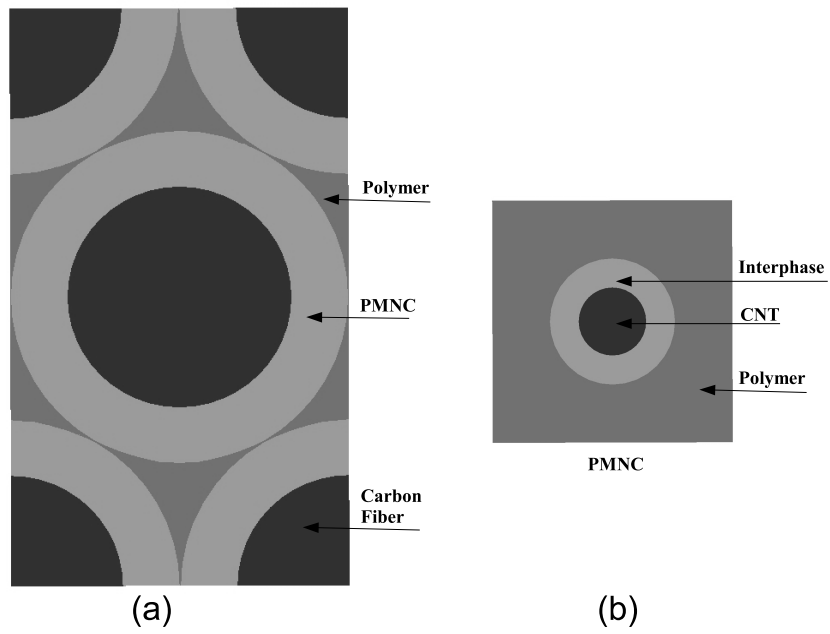


Figure 2: Representative volume elements: (a) Fuzzy fiber-reinforced composite, (b) Polymer matrix nanocomposite (PMNC).

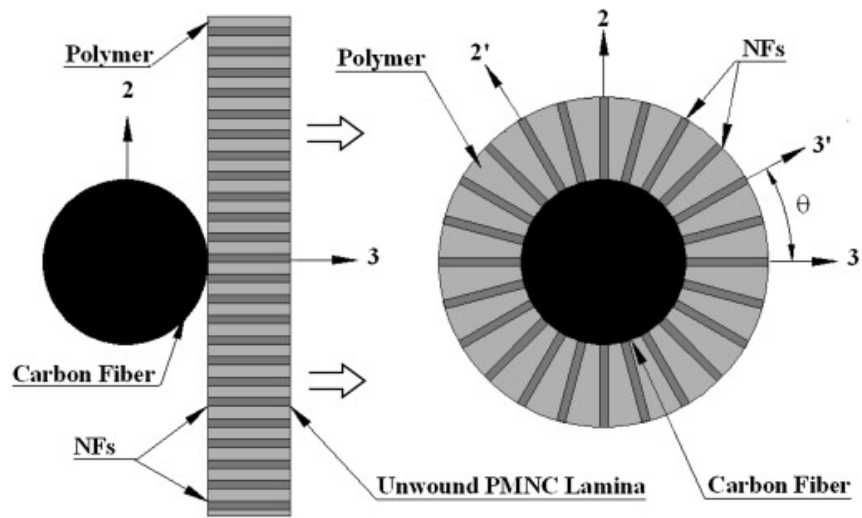


Figure 3: Unbound and bound polymer matrix nanocomposite with carbon fiber. NFs (nano-fiber: CNT + Interphase). Taken from [4].

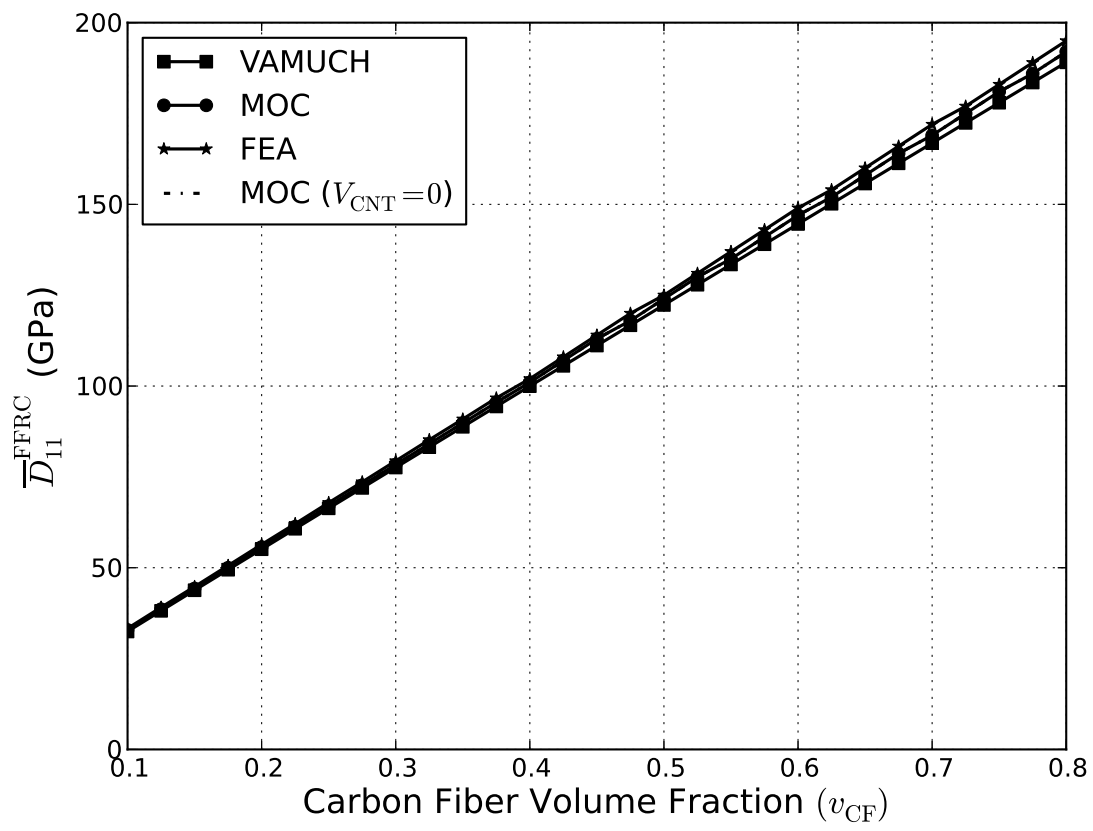


Figure 4: Variation of the effective elastic coefficient $\bar{D}_{11}^{\text{FFRC}}$ of the fuzzy fiber-reinforced composite with the carbon fiber volume fraction.

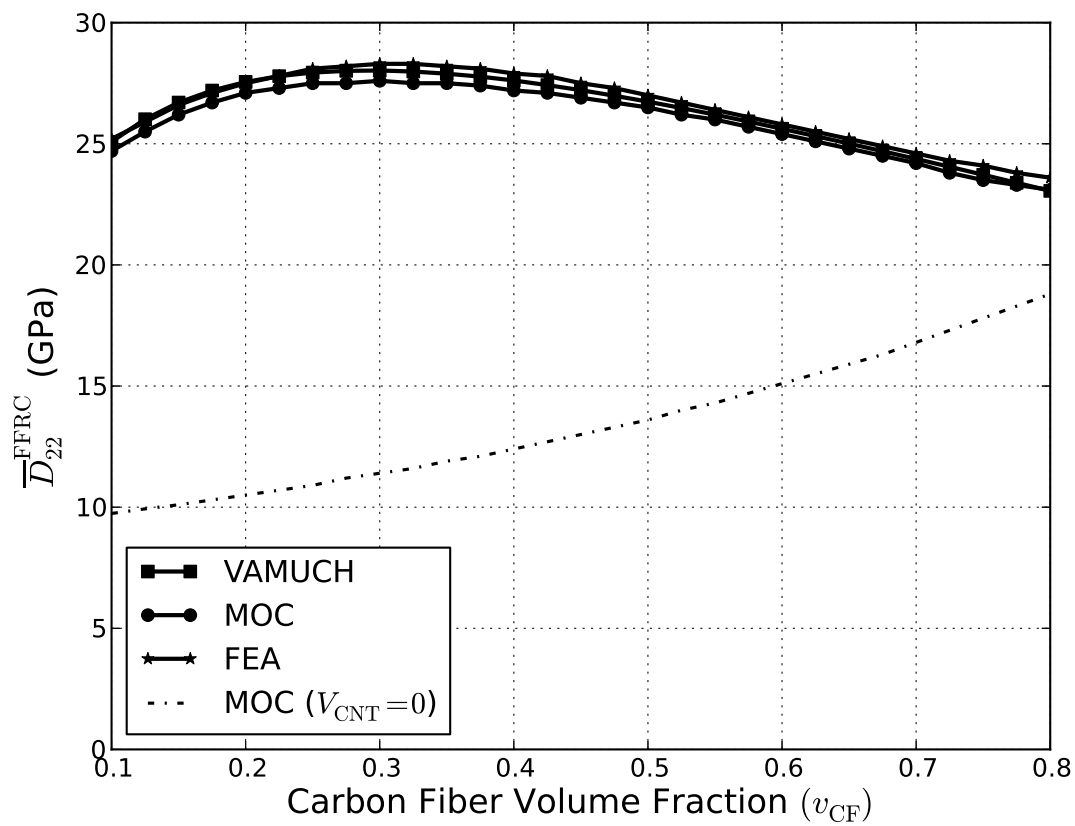


Figure 5: Variation of the effective elastic coefficient $\bar{D}_{22}^{\text{FFRC}}$ of the fuzzy fiber-reinforced composite with the carbon fiber volume fraction.

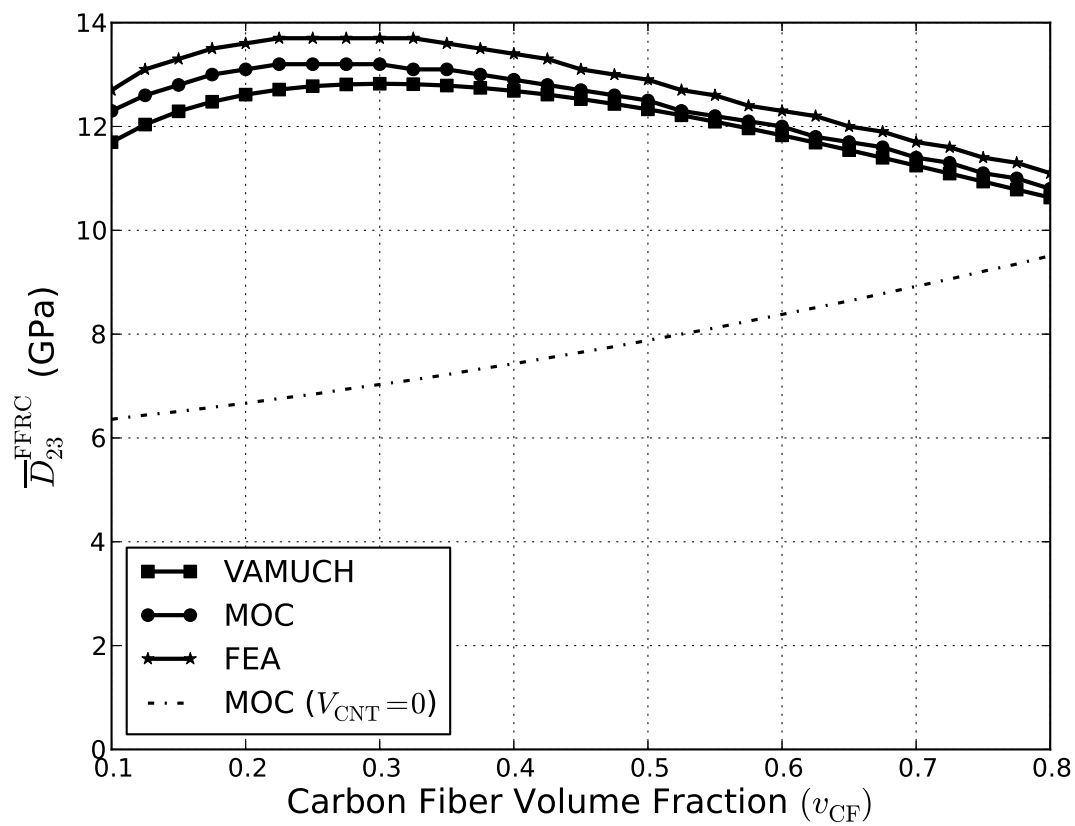


Figure 6: Variation of the effective elastic coefficient $\bar{D}_{23}^{\text{FFRC}}$ of the fuzzy fiber-reinforced composite with the carbon fiber volume fraction.

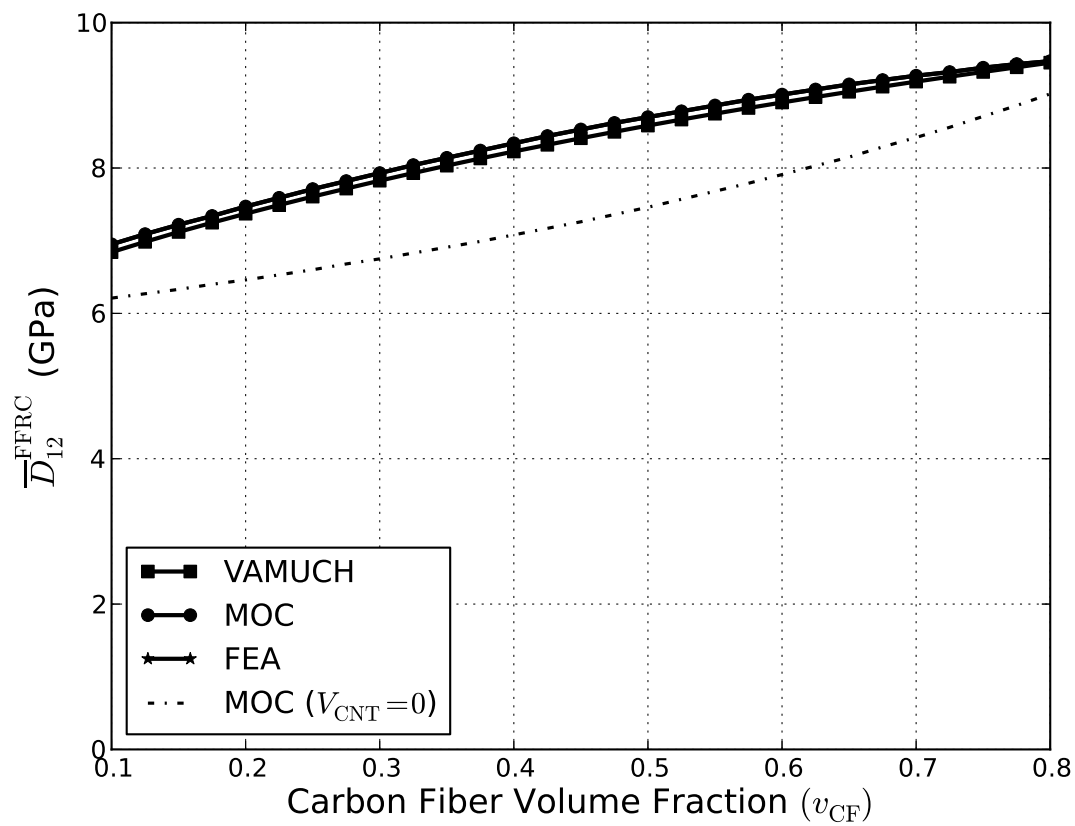


Figure 7: Variation of the effective elastic coefficient $\overline{D}_{12}^{\text{FFRC}}$ of the fuzzy fiber-reinforced composite with the carbon fiber volume fraction.

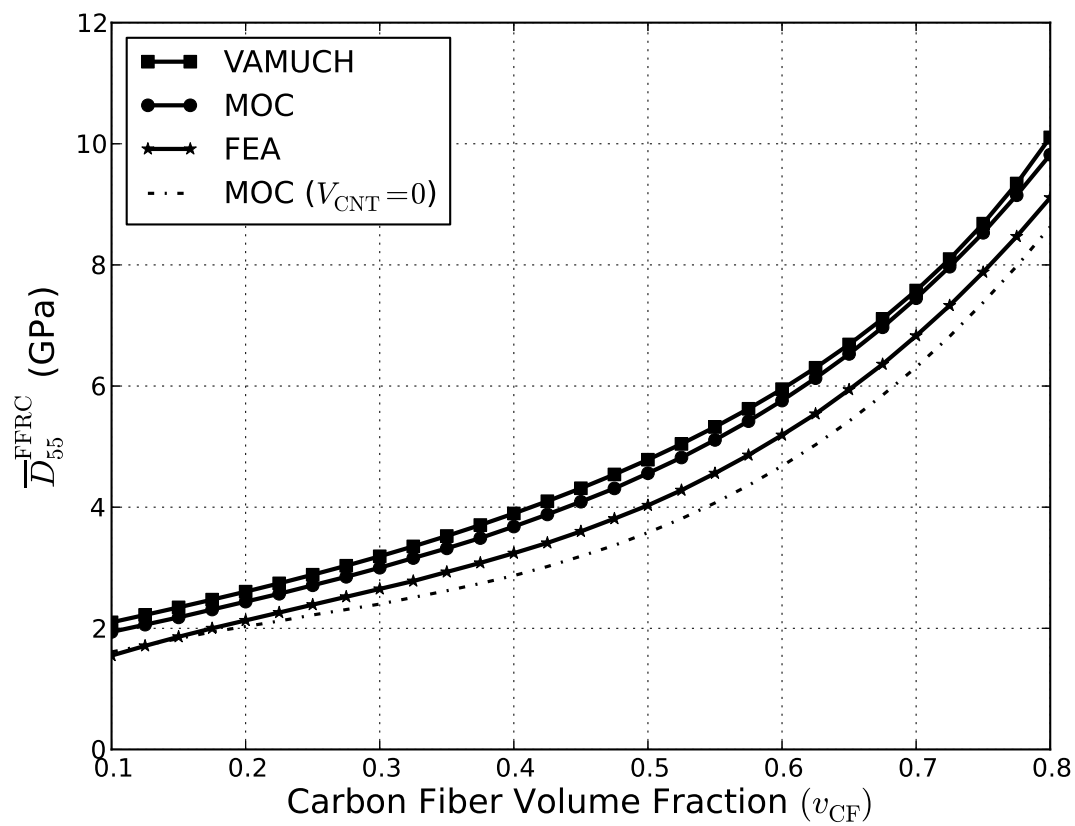


Figure 8: Variation of the effective elastic coefficient \bar{D}_{55}^{FFRC} of the fuzzy fiber-reinforced composite with the carbon fiber volume fraction.

Tables

Material	C_{11}	C_{12}	C_{13}	C_{23}	C_{33}	C_{44}	C_{55}
	[GPa]	[GPa]	[GPa]	[GPa]	[GPa]	[GPa]	[GPa]
(10, 0) CNT	709.9	172.4	240	240	1513.1	1120	268.7
Interphase	29.6	15.2	15.2	15.2	29.6	7.2	7.2
Carbon fiber	236.4	10.6	10.6	10.7	24.8	7	25
Polyimide	9	6	6	6	9	1.5	1.5

Table 1: Material properties of the constituent phases of fuzzy fiber-reinforced composite. Taken from [4].