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On the fatigue response of aluminium specimens reinforced with carbon epoxy patches

Marie-Laetitia Pastor*, Xavier Balandraud, Michel Grédiac, Jean-Louis Robert

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Fatigue tests were carried out on various types of aluminium specimens to characterize the beneficial effect of composite patches on undamaged and damaged aluminium substrates. The influence of both the machining process and the specimen geometry on the fatigue properties is first discussed. Undamaged and damaged specimens are patched and their fatigue strength is compared to their expected counterpart obtained with the $S-N$ curve of plain aluminium. It is shown that the shear strength of the adhesive strongly influences the fatigue response of the patched specimens, especially for the highest loading levels. The shear strength of the adhesive is obtained by coupling some quasi static tests and a suitable numerical model. When the maximum shear stress in adhesive is nearly equal to the shear strength, early failure of the specimens subjected to fatigue loading is observed. This clearly underlines the influence of this latter parameter on the fatigue response of patched structures.

Keywords: Aluminium; Composite patch; Fatigue life; Reinforcement; Transverse shear stress

1. Introduction

Composite patches are nowadays widely used to repair fatigue damaged metallic structures, especially in aeronautical industry [1,2]. These patches are generally bonded to bridge cracks which take place in thin metallic plates or shells such as fuselage skins damaged by fatigue loading. As a result, crack growth is delayed and the service life of the repaired structures is extended. The mechanical properties of such repaired structures have been studied in depth in various recent papers. Investigation into the influence of several parameters on the crack growth behaviour has been of primary focus of most of these studies. For instance, the influence of several parameters such as composite patch size [3], number of plies [4], asymmetry of the repaired structure [5], pretension of the overlays prior

to bonding [6], plasticity [7], imperfectly bonded composite patch [8], residual stresses [9] has been examined in the literature. A similar approach deals with reinforcement of metallic structures. In this case, the crack has not yet occurred in the metallic substrate and the objective is to bond a composite patch in such a way that crack initiation is delayed. The composite patch can be bonded away from the zone where the crack is expected to appear. In this case, the initial stress flow is deviated by the composite patch and the critical zone is relieved. Such an approach is expected to be efficient when one cannot bond a patch on the cracked zone, for instance if the crack appears in a very thick zone such as embedment of a wing in a fuselage. This technique has been recently studied in Refs. [10-12] for instance. It is clear that its efficiency strongly depends on the mechanical response of the bonding between metallic substrate and composite patch, especially under fatigue loading. To assess the fatigue response of a patched structure, a first approach is to consider that the fatigue response of the metallic substrate can be found by computing the stress state after reinforcement, for instance, in case

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of uniaxial loading, using a simple rule of three which takes into account the stiffness of both the substrate and the patch. The $S-N$ curve of the plain material then provides the response of the substrate in terms of fatigue life. This approach is however rather simple since some additional parameters may influence the response of the reinforced structure such as the fatigue strength of the composite itself or the response of the adhesive under shear fatigue loading. For a given type of adhesive, this mechanical response depends on various parameters such as surface preparation, defects or shape of the adhesive spew along the free edges of the patch [13]. Moreover, it is well known that a transverse shear stress peak as well as peeling stresses take place near the free edges of the composite patch. This local 3D stress state in the adhesive is also expected to influence the global fatigue response of the reinforced structure. Edge taper reduces the shear and peel stresses [14,15], but such a technical solution requires additional handling and seems difficult to use for patches exhibiting curved boundaries [12]. In this context, this study is aimed at studying the fatigue behaviour of a series of patched specimens under fatigue loading and to investigate the life extension due to the composite patches. A comparison with the lifetime predicted with the $S-N$ curve of plain aluminium is also performed. The main conclusion is that the lifetime extension can be reasonably predicted using the $S-N$ curve of plain aluminium if the adhesive actually sustains load transfer between substrate and patch up to aluminium failure.

2. Specimens and experiments

2.1. Introduction

Various types of specimens are tested in this study. First, the influence of cutting process on fatigue properties is examined. Then, the geometry of the patched specimens is investigated. Finally, the fatigue strength of patched specimens is compared with the fatigue strength of unpatched ones.

2.2. Comparison between laser cutting and water jet cutting

The specimen geometry is shown in Fig. 1. The gauge section exhibits a dog-bone shape, as specified in standards on aluminium testing [16]. Specimens are machined from unclad aluminium alloys 2024-T3 sheets. The specimens are 92 mm long, 25 mm wide (12 mm for the gauge section) and 3 mm thick. The length direction of all specimens is aligned with the rolling direction of the sheets, so the grain in material is along the length direction of the specimens. Such specimens made in plain aluminium are called “specimens A” hereafter. Particular attention is paid to the machining of the specimens. Previous studies on the influence of manufacturing on fatigue strength of metallic specimens clearly underline that the manufacturing process directly influences their fatigue response [17–19]. Two types

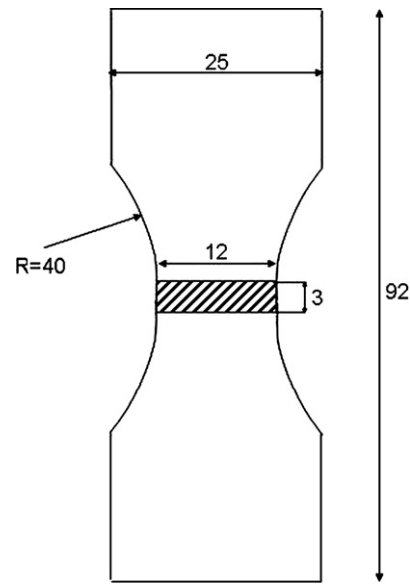


Fig. 1. Specimen A: plain aluminium.

of cutting processes are first investigated in the present study: laser cutting and water jet cutting. The $S-N$ curves under repeated load (stress ratio equal to zero) are plotted in Fig. 2 to highlight the influence of the type of machining process. A vertical shift of nearly 35 MPa can be observed. So the fatigue strength with water jet cutting is lower than the material fatigue strength with laser cutting. Finally, all specimens tested in the following are cut using water jet. The influence of heat affected zone which takes place near the edges is therefore avoided. It is also safer to consider results obtained with this type of process because fatigue strength obtained is lower. In addition, this process is widely spread in industry.

2.3. Influence of the composite patch geometry

3-ply unidirectional carbon epoxy composite patches are symmetrically bonded on both sides of the aluminium specimens. The composite is a T300-914 carbon epoxy prepreg system. The fibre direction is aligned with the principal axis of the specimens. The composite patches are bonded on the specimens with a nominal 0.1 mm thick layer of bismaleimide Redux 312 film adhesive supplied by Hexcel. Specimens are allowed to cure at 120 °C for 1 h and 0.3 MPa in a platen press. The mechanical properties of the aluminium, the composite and the adhesive are given by the suppliers. These quantities are collected in Table 1. Fatigue tests are conducted on a 100 kN MTS testing machine. A 10 Hz sinusoidal tensile loading is applied during the tests.

Two types of reinforced specimens are first tested to investigate the influence of their geometry on the fatigue response. In the first case (Specimen B in Fig. 3), the patch is rectangular and bonded in such a way that its width is nearly equal to the minimum width of the specimen. In the second case (Specimen C in Fig. 3), the composite is

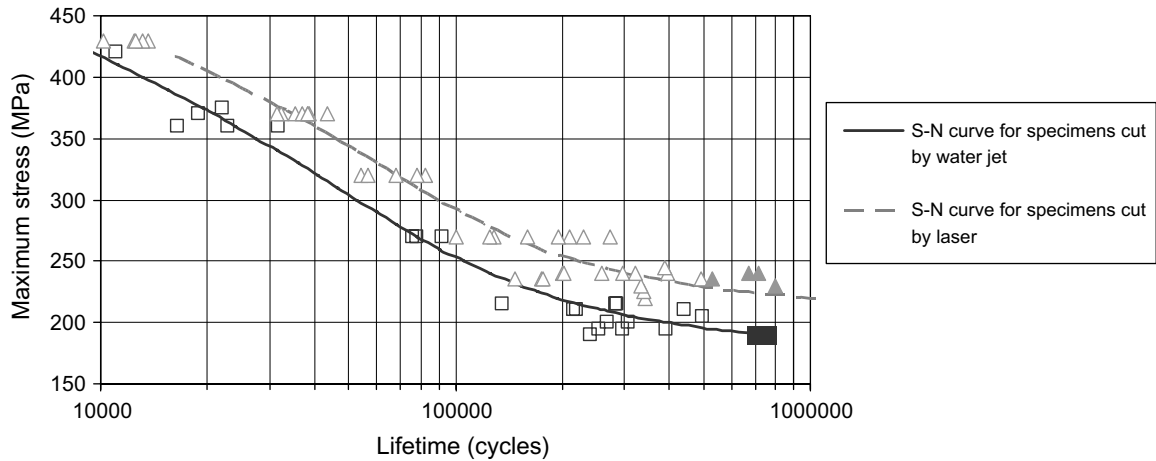


Fig. 2. *S N* curves for specimens cut by laser/water jet.

Table 1
Mechanical properties of the materials involved in the reinforced specimens

Aluminium	$E=73.1$ GPa, ν 0.34
Composite	E_{11} 130.5 GPa, E_{22} 10 GPa, ν_{12} 0.28, G_{12} 7 GPa
Adhesive	$E=4.2$ GPa, ν 0.30

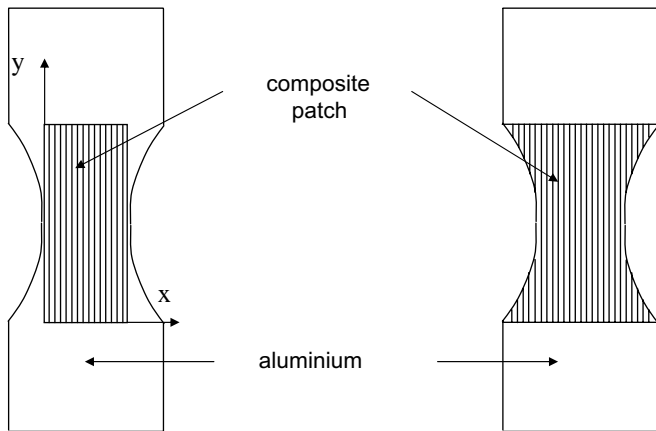


Fig. 3. Two types of reinforced specimens.

bonded on the aluminium sheet prior to machining the specimens. The objective here is to detect possible differences that could be caused by the shape and/or the milling of the composite patch. It must be pointed out that specimens A are exposed to the same temperature cycle as the reinforced specimens (120 °C for 1 h). So the aluminium substrates of specimens B and C after composite curing are assumed to be in the same metallurgical state as specimens A.

The loading applied to specimens A is lower than the material yield stress in the gauge section (345 MPa), but greater than the fatigue strength at 500,000 cycles which is equal to 186 MPa. Specimens B and C are then subjected to the same applied loading, thus leading to a lower stress in the aluminium gauge section.

Fig. 4 shows three typical specimens after failure. As expected, failure always occurs in the gauge section of specimens A. On the contrary, failure does not exactly occur in the gauge section of the two types of reinforced specimens. For specimens B, the composite patch separates from the substrate, thus indicating a failure of adhesion to the substrate. For specimens C, transverse failure of the composite patch also occurs along two parallel lines while debonding from aluminium substrate.

Table 2 and Fig. 5 show the results obtained in terms of lifetime of the specimens. First it can be seen that specimens A exhibit the same lifetime as the specimens used for constructing the *S N* curve of plain aluminium. These latter were not heated prior to testing. This feature clearly indicates that the heat treatment applied to the aluminium sheets while bonding the composite patches does not affect its response in terms of fatigue properties. As expected, specimens B and C actually exhibit a longer lifetime than

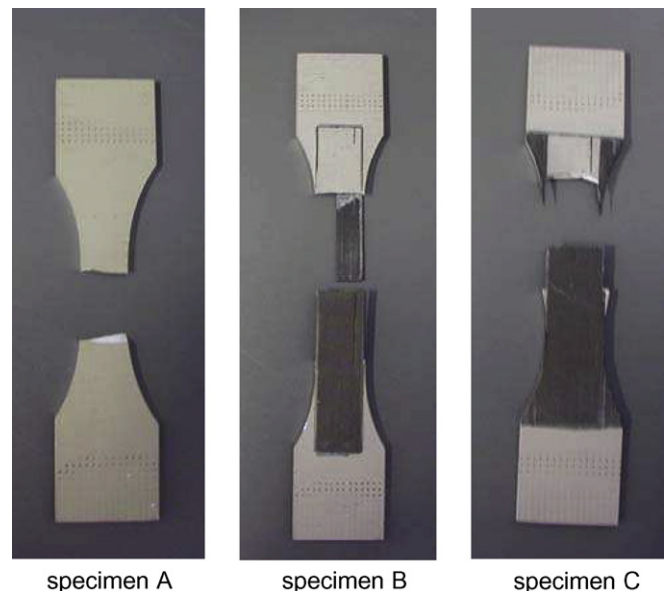


Fig. 4. Specimens after failure.

Table 2
Lifetime of the three types of specimens

Specimen	Mean lifetime (cycles)	Standard deviation (cycles)
A	41,000	4400
B	385,780	150,346
C	292,118	35,745

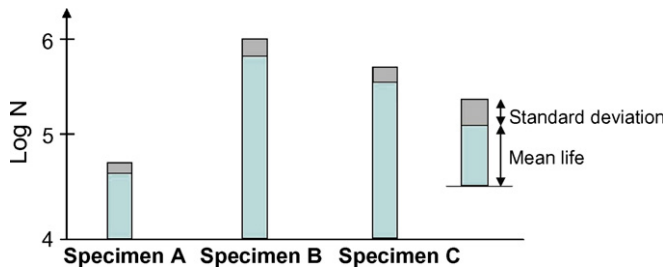


Fig. 5. Number of cycles at failure and standard deviation of specimens A, B, and C.

specimens A. Since the applied loading is the same in the three cases, it is clear that the tensile stress within aluminium is lower in the two last cases, thus leading to an increase of fatigue life. Another interesting result is the fact that specimens B exhibit a greater lifetime than specimens C. This shows that the shape of the patch as well as the machining process influence the response of the specimens although the gauge section is the same in both cases. A possible reason is the fact that milling induces some fibre pull-out. The ratio between standard deviation and mean lifetime is higher for specimens B and C than for specimens A. This is probably due to the fact that those specimens are more complex. This feature induces much more variability in the geometry, especially because of the adhesive thickness which is not actually a constant over the specimen. As the mean lifetime is lower for specimens C, it was decided to prepare and to test only specimens of type B in the remaining part of the study.

3. Fatigue tests on specimens B

3.1. Introduction

A series of 30 specimens of type B is prepared and tested. The substrate of ten of these specimens is made in plain aluminium. Ten other ones are subjected to a fatigue test up to 25% of the fatigue life before patching. The last ten specimens are subjected to a fatigue test up to 40% of the fatigue life before patching. The objective here is to assess the actual effect of reinforcement on specimens subjected to initial fatigue damage. This situation is encountered in real structures since damaged structures (and not undamaged structures) are reinforced in practice. It must be pointed out that the scatter of fatigue results obtained with different aluminium plates is significant. As a result, it was decided to use a $S-N$ curve obtained with a set of specimens cut in the same plate as that used for the patched specimens tested below.

The loading is such that two different cases of maximum longitudinal stress take place in the aluminium gauge section: $\sigma_s = 195$ and 280 MPa. $\sigma_s = 195$ MPa is just above the fatigue strength at 5×10^5 cycles whereas $\sigma_s = 280$ MPa is nearly at the middle of the $S-N$ curve. These values are deduced from the applied loading F by studying the equilibrium of a patched section away from the free boundary of the patch. It is a trivial matter to obtain the stress level in substrate in the gauge section

$$\sigma_s = \frac{F}{S_s \left(1 + \frac{E_p S_p}{E_s S_s} \right)} \quad (1)$$

where S_s and S_p are the section of the aluminium substrate and patches, respectively.

The aim is now to examine which of the three materials (aluminium, adhesive or composite) fails first and eventually to assess the actual beneficial effect of patching in terms of fatigue life extension.

3.2. Results

3.2.1. Undamaged patched specimens

Experimental results found with the specimens are collected in Fig. 6a. To highlight the effect of patching, the stress reported in this figure is not the stress in the gauge section of reinforced specimens, but the stress that would take place in aluminium for the same applied loading and without any patch. This is also justified because reinforced and unreinforced structures sustain the same global loading in practice. As a result, 195 MPa in the aluminium of a patched specimen is equivalent to 312 MPa in a unpatched specimen and 280 MPa is equivalent to 446 MPa. This fictitious stress is called *equivalent stress* in the following.

The main result is the fact that the life extension is all the higher as the stress is smaller. This life extension can be measured by the ratio between the fatigue life of patched specimens divided by the fatigue life of plain aluminium specimens subjected to the same loading. This ratio is referred to as *life extension ratio* in the following. The life extension ratio at 195 MPa is 8.4 whereas it is equal to 2.5 at 280 MPa.

3.2.2. Damaged specimens

It has first been checked with some preliminary tests that the heat treatment on damaged specimens does not modify the fatigue response of the aluminium substrate. Results obtained with damaged specimens are reported in Fig. 6b and c. The $S-N$ curves in these figures are horizontally shifted from the above one since the *residual* fatigue life (and not the *total* fatigue life) is reported here. This residual fatigue life is equal to 75% and 60% of the total life in the two cases under study.

As can be seen in Fig. 6b and c, the lifetime is greater with reinforced damaged specimens than with undamaged specimens. In the first case of reinforced damaged speci-

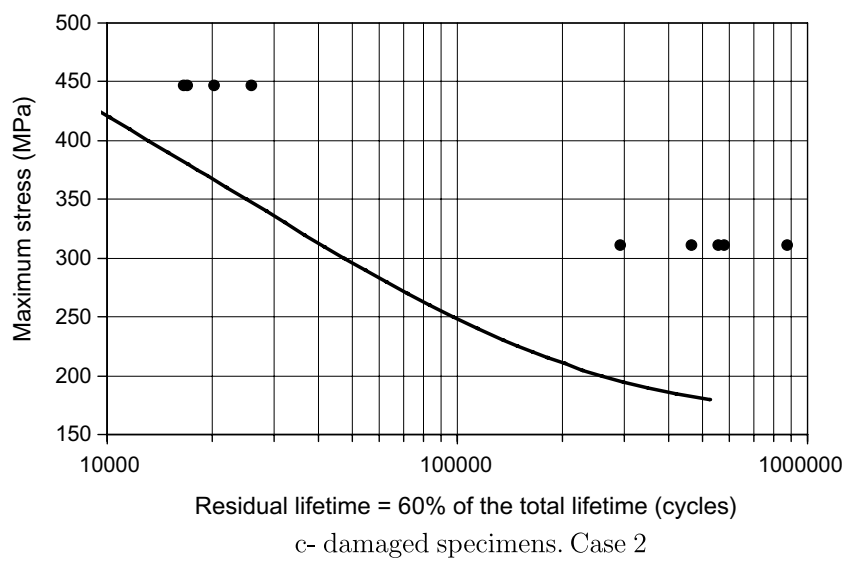
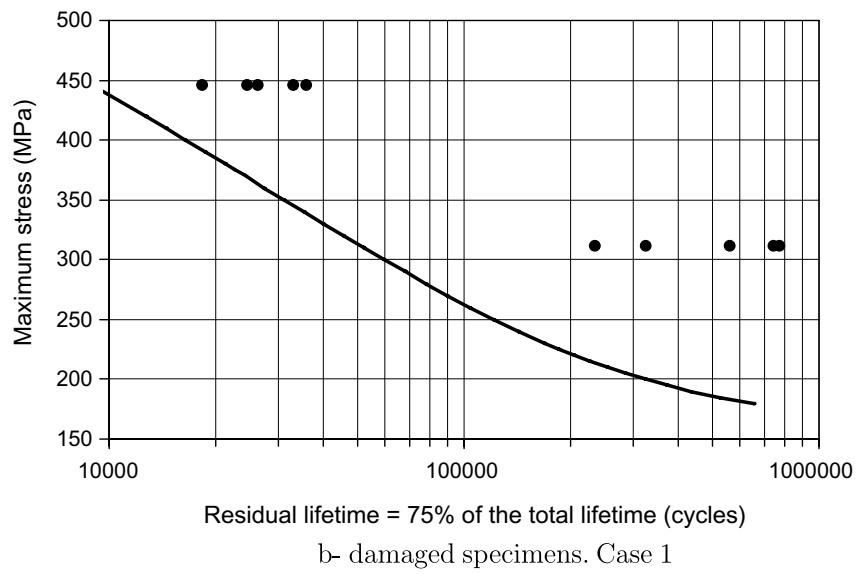
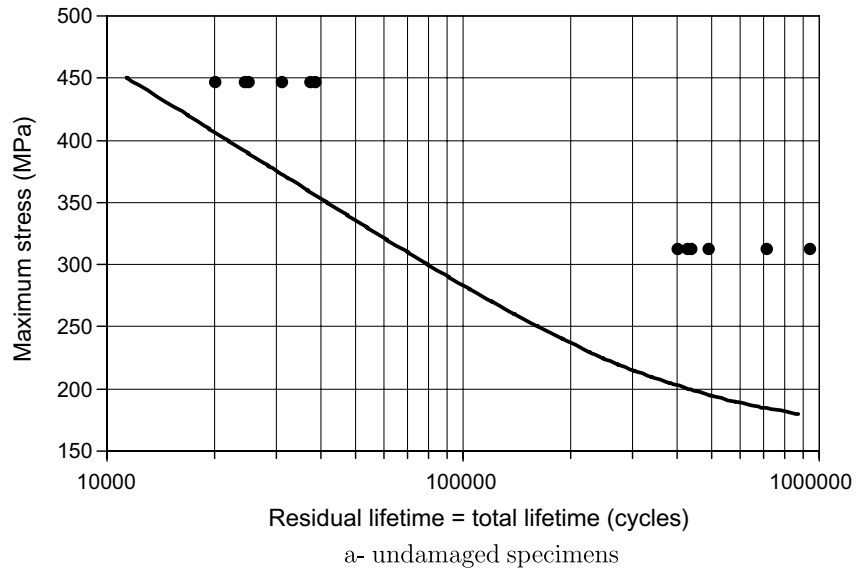


Fig. 6. Fatigue strength of patched specimens with different initial damage levels.

mens, the residual life extension ratio is 10.3 and 3.1 at 195 and 280 MPa, respectively. In the second case, it is equal to 11.2 and 2.8 at 195 and 280 MPa, respectively, thus clearly highlighting the beneficial effect of reinforcement on damaged structures. The fatigue life extension is lower at 280 MPa than at 195 MPa. The reason is the failure of the adhesive which occurs at an earlier stage in the first case. Let us now investigate this issue in more detail.

3.2.3. Effect of the mechanical response of the adhesive

Fig. 7a and b represents two typical curves of the relative displacement while testing. This relative displacement is defined by the elongation of the specimen from the minimum to the maximum load during one cycle. Any softening of the joint due to damage or crack appearance in this part of the specimen is detected by an increasing relative displacement. This quantity clearly increases while testing at 280 MPa whereas it fairly remains constant at

195 MPa. This underlines the progressive damage of the adhesive (see Fig. 7b). This damage causes an increasing stress level in the aluminium substrate from about one third of the test onwards. The adhesive between one of the patch and the substrate suddenly fails and the relative displacement sharply increases. The stress level in the whole aluminium substrate also suddenly increases at the same time. This finally causes early failure of the substrate compared to the expected lifetime given by the $S-N$ curve. This phenomenon can be observed in Fig. 8a c where the actual stress level in the aluminium substrate is superimposed in the three cases studied in Section 3.2.1. These previous experimental points are in fact simply shifted downwards. In all cases, experimental points at 195 MPa are in good agreement with the expected lifetime given by the $S-N$ curve of plain aluminium. On the contrary, experimental points at 280 MPa illustrate early failure of the specimens with respect to the lifetime predicted from the

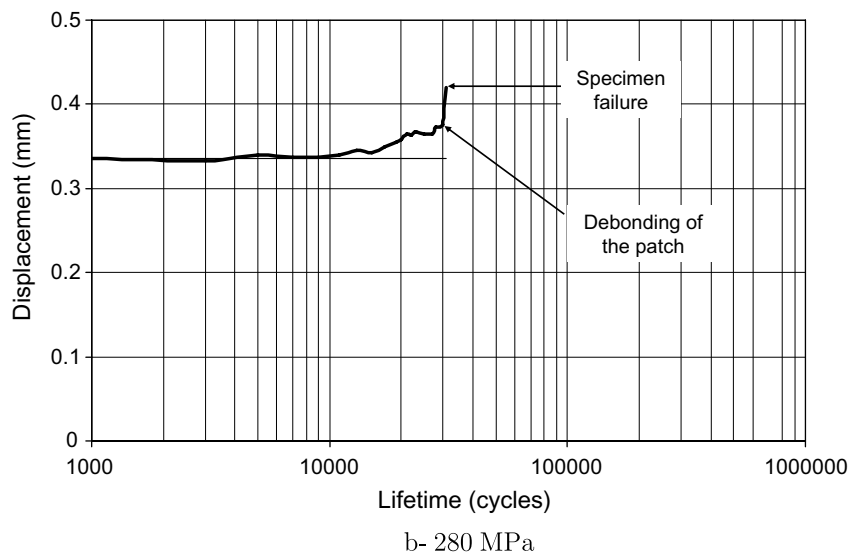
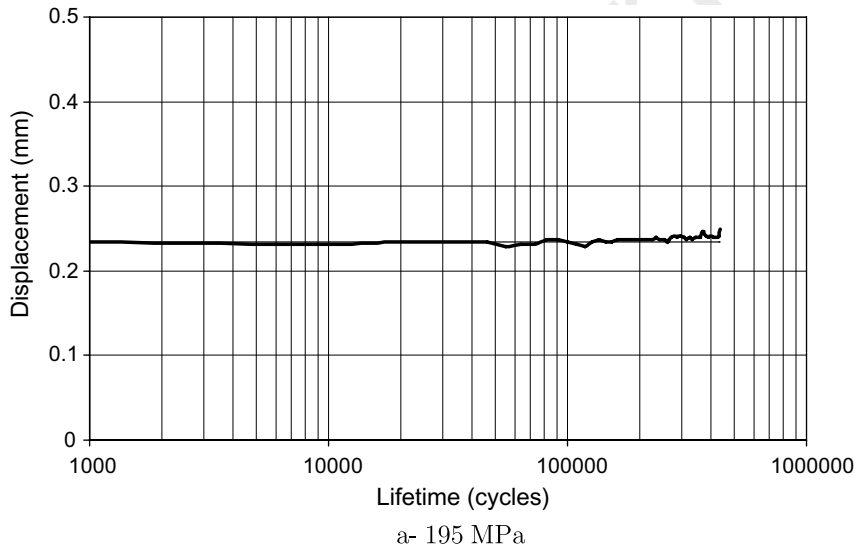


Fig. 7. Elongation while testing.

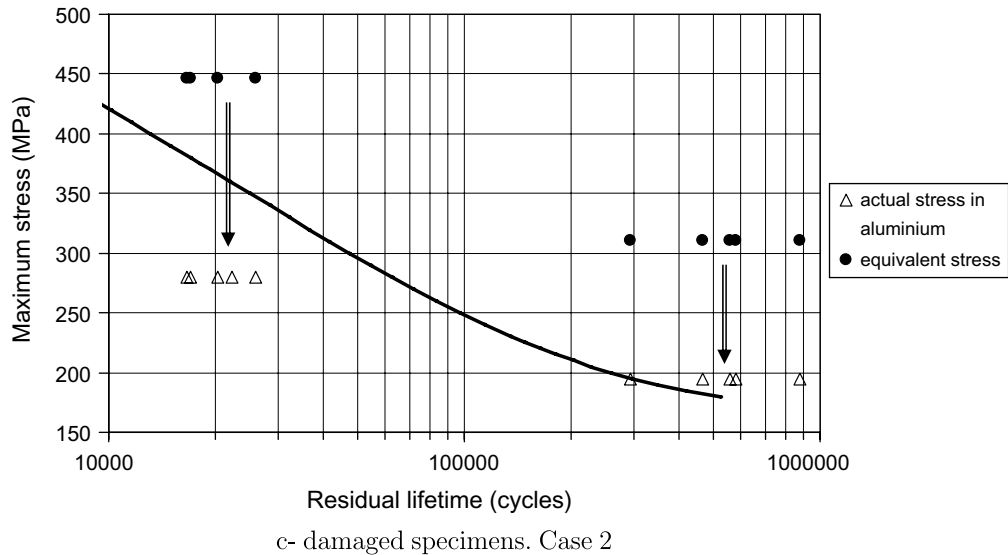
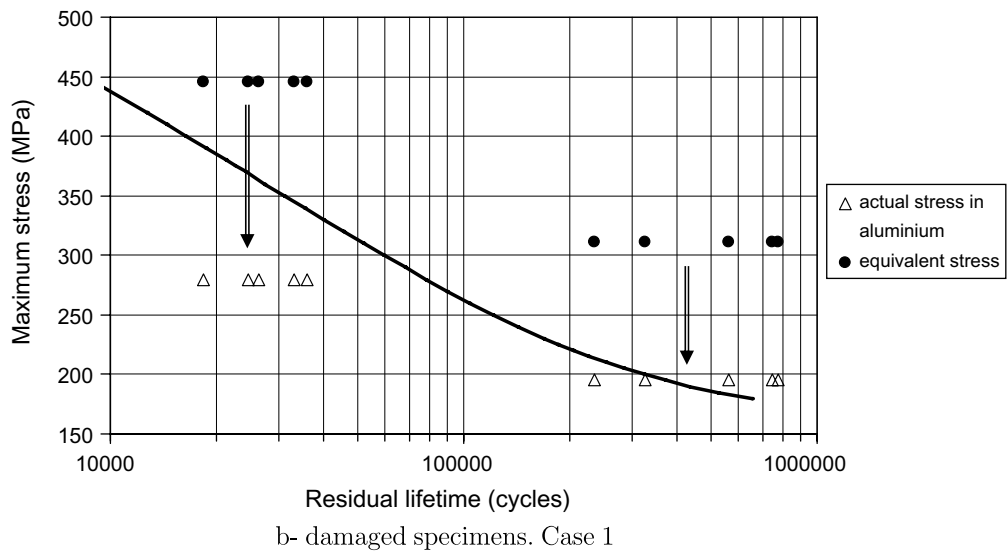
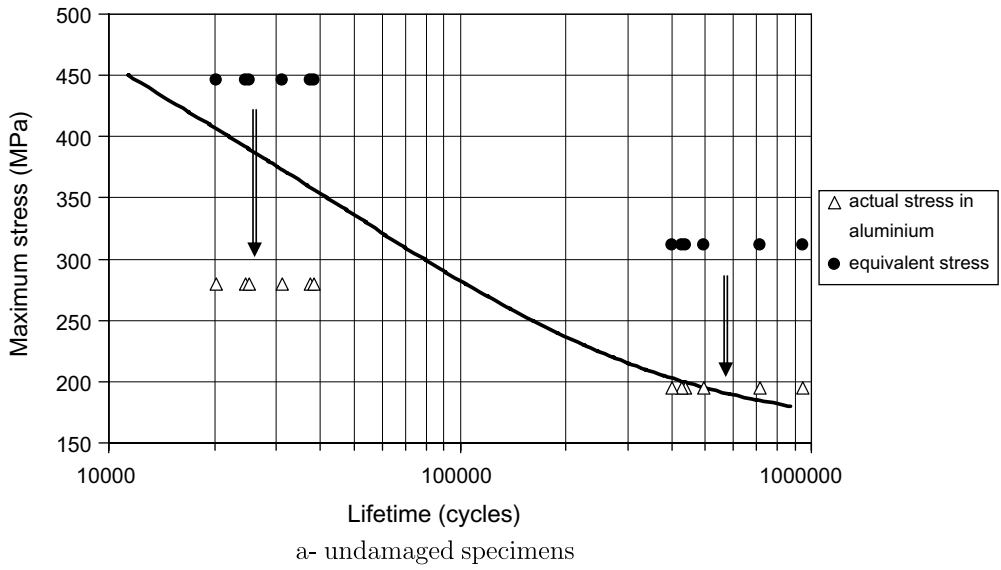


Fig. 8. Fatigue strength of patched specimens with different initial damage levels. Actual stress in aluminium substrate.

$S-N$ curve of aluminium. This lends credence to the failure scenario proposed above.

3.2.4. Shear strength of the adhesive

It is finally interesting to estimate the transverse shear stress in the adhesive in the two loading cases (195 and 280 MPa) and to compare these quantities to the shear strength of the adhesive. The transverse shear stress mainly takes place near the free boundary of the patch. Calculating the shear stress distribution in this zone can be carried out using the so-called “shear-lag” model [20] for instance, but this simple model is valid only if the patched specimen exhibits constant width and thickness. This model is not suitable in the present case since the section of the specimen is not a constant. This shear stress distribution near the free edge of the patch could be also computed using a finite ele-

ment model, but it is well known that only 3D models with very refined meshes may lead to accurate results in terms of shear stress in the adhesive near the free boundary. Such very heavy calculations should be avoided. So a specific procedure based on both a 2D finite element calculation and a finite difference is presently used. First, a 2D model of the patched specimen enables to deduce the in-plane stress components in the aluminium substrate as well as in the patch. 18,712 four-noded Shell 63 elements of the ANSYS package are used for this purpose. Stresses in the patch and in the substrate provided by the model are then considered as input data of a finite difference scheme which finally provides the transverse shear distribution in the adhesive. This numerical procedure is not described here. Full details can be found in Ref. [21].

The mesh of the specimen used in the 2D finite element model is shown in Fig. 9. The rectangular shape of the patch can be recognized in this figure. The normal stress distribution in the patch σ_p and the transverse shear stress distribution in the adhesive τ_a are shown in Figs. 10 and 11, respectively. As can be seen, a shear stress peak takes place near the free boundary of the patch. The amplitude is about 60 MPa for an applied loading equal to 11,000 N. The corresponding normal stress in aluminium is 195 MPa. If this normal stress is equal to 280 MPa, the transverse shear stress reaches 86 MPa. It is of prime importance to compare these estimates of the shear stress in the adhesive with its static shear strength.

The correct determination of the static shear strength of the adhesive is an important issue since this value strongly depends on various parameters among which the manufacturing process or unavoidable geometrical defects. It was therefore decided to directly measure this quantity on some specimens similar to those used for the fatigue tests. Four specimens were subjected to quasi-static tensile tests to determine this quantity. These specimens were equipped with FLG-02-23 Tokyo Sokki Kenkyujo strain gauges bonded on the composite patches, 3 mm away from the free boundary (see Fig. 12). It was then possible to detect

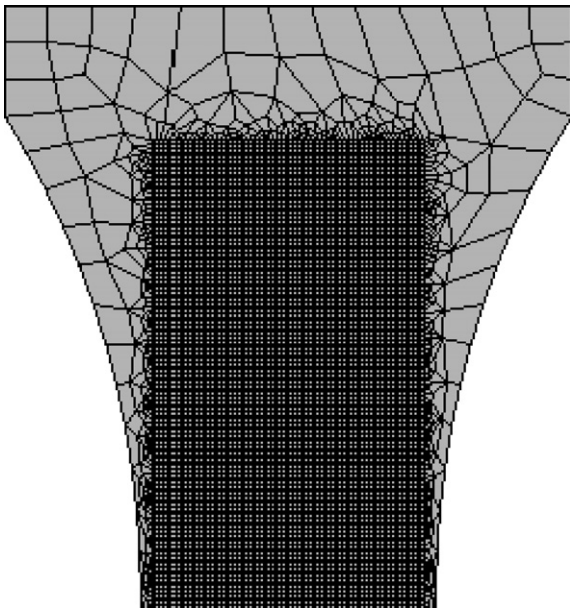


Fig. 9. Mesh of the upper half of the patched specimen.

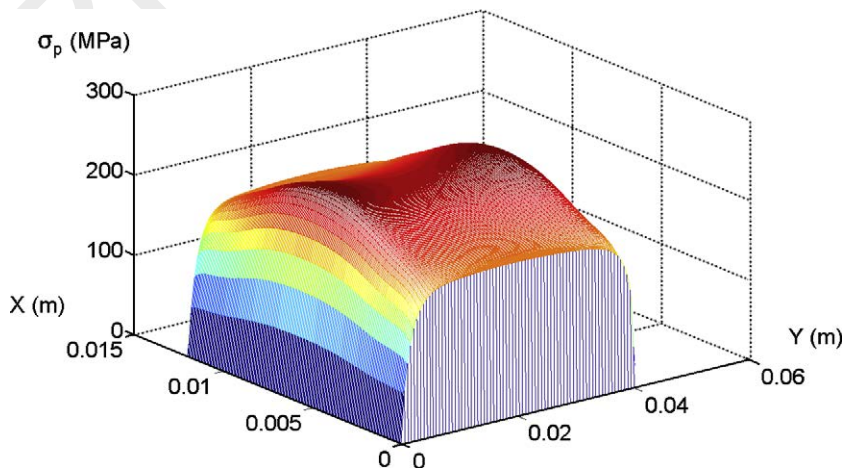


Fig. 10. Normal stress distribution σ_p in the composite patch.

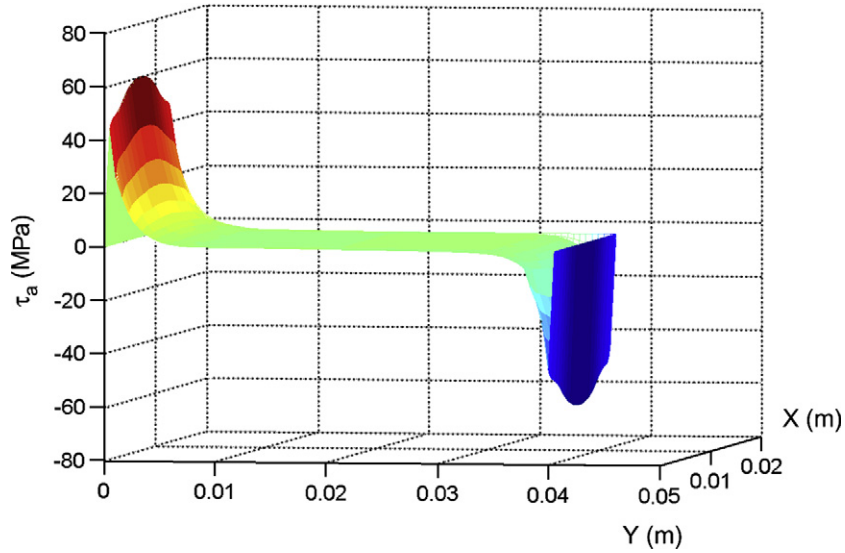


Fig. 11. Transverse shear distribution τ_a in the adhesive layer.

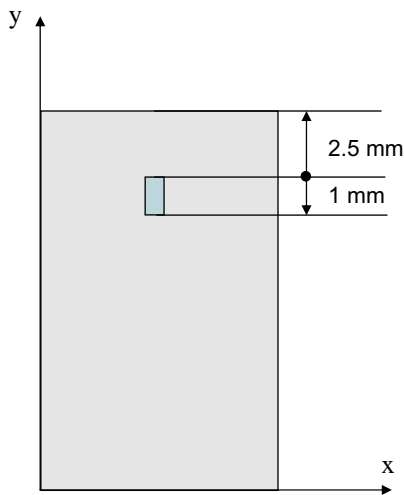


Fig. 12. Schematic view of the gauge bonded on the composite patch.

very accurately the failure of the bonded joint since this failure was expected to initiate in this zone. Two hand screw clamps were affixed at the bottom of the composite patch to avoid debonding of the bottom part of the specimens. So only two strain gauges were bonded on the composite patches: one at the upper part of each patch. Fig. 13 shows a typical strain force curve obtained during one of these tests. The longitudinal strain is measured with the strain gauge whereas F is the global force applied to the specimen. The longitudinal strain increases as a fairly linear rate up to failure of the joint. The corresponding shear stress in the adhesive is then deduced from the maximum longitudinal strain measured during the tests using the numerical model described above. The shear strength obtained for each specimen is reported in Table 3. As can be seen, the scatter of results is rather low. The shear stress in the adhesive for a normal stress in aluminium of 280 MPa is nearly equal to this shear strength. This is prob-

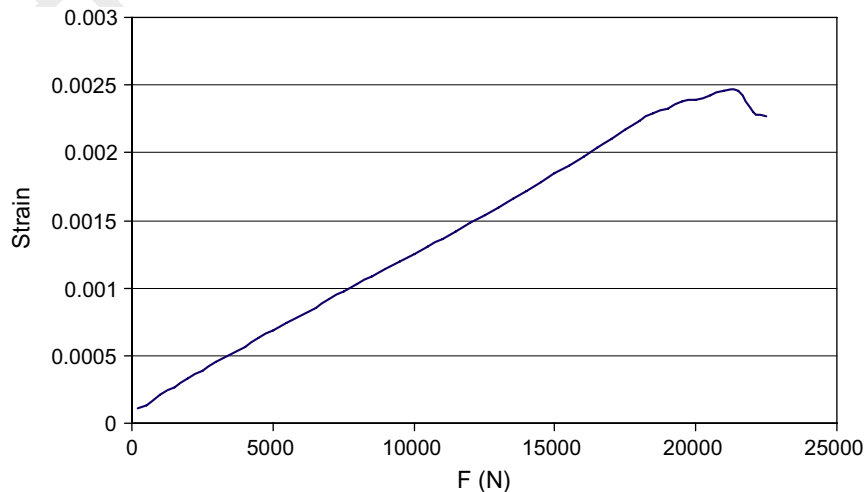


Fig. 13. Longitudinal strain vs. applied loading.

Table 3
Shear strength of the adhesive

Specimen	1	2	3	4	Average	Standard deviation
Shear strength (MPa)	106	109	90	98	101	3.7

ably the reason why the fatigue life extension is lower at 280 MPa than at 195 MPa. In this latter case, it is interesting to note that the most likely failure scenario is that aluminium substrate fails before adhesive, thus meaning that adhesive fulfills the requirement of a safe and reliable joint between patch and damaged substrate. From an industrial point of view, particular attention has to be paid on the correct determination of the actual stress distribution in adhesive since it is here clearly illustrated that this quantity governs the failure of the reinforced structures beyond a certain value which seems to be close to the static shear strength.

4. Conclusion

The fatigue behaviour of aluminium specimens reinforced with composite patches was investigated in this study. The following conclusions can be drawn from the experimental results obtained:

- The lifetime extension of the reinforced specimens is significant assuming the same loading level between patched and unpatched specimens: this quantity lies between 8 and 11 for an equivalent stress close to the fatigue strength up to 5×10^5 cycles of plain aluminium and reaches three when the shear stress in adhesive is close to the corresponding static shear strength.
- The lifetime extension clearly depends on the actual damage level in substrate as it is reinforced.
- Tested specimens exhibit different failure scenarios which depend on the loading level: either the aluminium or the adhesive first fails. In the first case, the lifetime extension is reasonably predicted with the $S-N$ curve of plain aluminium. In this second case, the shear stress in adhesive is close to its shear strength. After patch debonding, the stress in aluminium suddenly increases and failure then promptly occurs, thus shortening the lifetime extension of the reinforced coupon given by the $S-N$ curve.
- It would be interesting to perform similar tests with composite patches exhibiting edge tapers to check whether the fatigue strength of such patched structures is enhanced at the highest loading levels.

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